RELATIONS AND FUNCTIONS

1. The equivalent definition of $f(x) = \left||x| - 1\right|$, is

a) $f(x) = \begin{cases} -x-1, & x \le -1 \\ x+1, & -1 < x \le 0 \\ 1-x, & 0 \le x \le 1 \\ x-1, & x \ge 1 \end{cases}$

b) $f(x) = \begin{cases} x-1, & x \le -1 \\ x+1, & -1 < x \le 0 \\ x-1, & 0 \le x \le 1 \\ x+1, & x \ge 1 \end{cases}$

c) $f(x) = \begin{cases} x+1, & x \ge 0 \\ x+1, & x \le 0 \end{cases}$

d) None of these

2. The domain of definition of $f(x) = \log_{100x}\left(\frac{2\log_{10}x+1}{-x}\right)$, is

a) $(0,10^{-2}) \cup (10^{-2}, 10^{-1/2})$

b) $(0, 10^{-1/2})$

c) $(0,10^{-1})$

d) None of these

3. The domain of the function

$f(x) = \frac{\sin^1(x-3)}{\sqrt{9-x^2}}$ is

a) $[2, 3]$ b) $[2, 3)$ c) $[1, 2]$ d) $[1, 2)$

4. If R denotes the set of all real numbers, then the function $f: R \to R$ defined by $f(x) = |x|$ is

a) One-one only b) Onto only

c) Both one-one and onto d) Neither one-one nor onto

5. If $f(x) = \frac{1}{\sqrt{-x}}$, then domain of fof is

a) $(0, \infty)$ b) $(-\infty, 0)$ c) $\{0\}$ d) $\{\}$

6. Let f be a real valued function with domain R such that

$f(x+1) + f(x-1) = \sqrt{2}f(x)$ for all $x \in R$, then,

a) $f(x)$ is a periodic function with period 8

b) $f(x)$ is a periodic function with period 12

c) $f(x)$ is a non-periodic function

d) $f(x)$ is a periodic function with indeterminate period

7. If D_{30} is the set of the divisors of 30, $x, y \in D_{30}$, we define $x + y = \text{LCM}(x, y), x.y = \text{GCD}(x, y), x' = \frac{30}{x}$ and $f(x, y, z) = (x+y).(y'+z)$, then $f(2, 5, 15)$ is equal to

a) 2 b) 5 c) 10 d) 15

8. The domain of definition of the function

$f(x) = \sqrt{\log_{10}\left(\frac{5x-x^2}{4}\right)}$ is

a) $[1, 4]$ b) $[1, 0]$ c) $[0, 5]$ d) $[5, 0]$

9. Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, then which of the following relations is a function from A to B?

a) $\{(1,2),(2,3),(3,4),(2,2)\}$ b) $\{(1,2),(2,3),(1,3)\}$
c) $\{(1,3),(2,3),(3,3)\}$ d) $\{(1,1),(2,3),(3,4)\}$

10. Let $f: R \to R, g: R \to R$ be two functions given by $f(x) = 2x - 3, g(x) = x^3 + 5$. Then, $(fog)^{-1}x$ is equal to
a) $\left(\frac{x-7}{2}\right)^{1/3}$ b) $\left(\frac{x+7}{2}\right)^{1/3}$ c) $\left(x - \frac{7}{2}\right)^{1/3}$ d) $\left(\frac{x-2}{7}\right)^{1/3}$

11. Let $f : [\pi, 3\pi/2] \to R$ be a function given by
$f(x) = [\sin x] + [1 + \sin x] + [2 + \sin x]$
Then, the range of $f(x)$ is
a) $\{0, 3\}$ b) $\{1\}$ c) $\{0, 2\}$ d) $\{3\}$

12. If the functions $f(x) = \log(x-2) - \log(x-3)$ and $g(x) = \log\left(\frac{x-2}{x-3}\right)$ are identical, then
a) $x \in [2, 3]$ b) $x \in [2, \infty)$ c) $x \in (3, \infty)$ d) $x \in R$

13. If D is the set of all real x such that $1 - e^{\frac{1}{x}-1}$ is positive, then D is equal to
a) $(-\infty, 1]$ b) $(-\infty, 0)$ c) $(1, \infty)$ d) $(-\infty, 0) \cup (1, \infty)$

14. Let $f(x) = \frac{\propto x^2}{x+1}, x \neq -1$. The value of $\propto$ for which $f(a) = a, (a \neq 0)$ is
a) $1 - \frac{1}{a}$ b) $\frac{1}{a}$ c) $1 + \frac{1}{a}$ d) $\frac{1}{a} - 1$

15. Let $f(x)$ be defined on $[-2,2]$ and is given by
$f(x) = \begin{cases} -1, -2 \leq x \leq 0 \\ x - 1, 0 < x \leq 2 \end{cases}$
and $g(x) = f(|x|) + |f(x)|$. Then, $g(x)$ is equal to
a) $\begin{cases} -x, & -2 \leq x < 0 \\ 0, & 0 \leq x < 1 \\ x - 1, & 1 \leq x \leq 2 \end{cases}$
b) $\begin{cases} -x, & -2 \leq x < 0 \\ 0, & 0 \leq x < 1 \\ 2(x - 1), & 1 \leq x \leq 2 \end{cases}$
c) $\begin{cases} -x, & -2 \leq x < 0 \\ x - 1, & 0 \leq x \leq 2 \end{cases}$
d) None of these

16. If $f: R \to R$ and $g: R \to R$ are defined by $f(x) = x - 3$ and $g(x) = x^2 + 1$, then the values of x for which $g\{f(x)\} = 10$ are
a) $0, -6$ b) $2, -2$ c) $1, -1$ d) $0, 6$

17. If $f: R \to R$ and $g: R \to R$ are defined by $f(x) = 2x + 3$ and $g(x) = x^2 + 7$, then the values of x such that $g(f(x)) = 8$ are
a) $1, 2$ b) $-1, 2$ c) $-1, -2$ d) $1, -2$

18. The domain of the real function $f(x) = \frac{1}{\sqrt{4-x^2}}$ is
a) The set of all real numbers b) The set of all positive real numbers
c) (-2, 2) d) [-2, 2]

19. If $f(0) = 1, f(1) = 5, f(2) = 11$, then the equation of polynomial of degree two is
a) $x^2 + 1 = 0$ b) $x^2 + 3x + 1 = 0$ c) $x^2 - 2x + 1 = 0$ d) None of these

20. If $[x]$ and $\{x\}$ represent integral and fractional parts of x, then the expression $[x] + \sum_{r=1}^{2000} \frac{\{x+r\}}{2000}$ is equal to
a) $\frac{2001}{2}x$ b) $x + 2001$ c) x d) $[x] + \frac{2001}{2}$

21. Suppose $f : [-2, 2] \to R$ is defined by
$f(x) = \begin{cases} -1 \text{ for } -2 \leq x \leq 0 \\ x - 1 \text{ for } 0 \leq x \leq 2 \end{cases}$
then $\{x \in [-2, 2] : x \leq 0 \text{ and } f(|x|) = x\} =$
a) $\{-1\}$ b) $\{0\}$ c) $\{-1/2\}$ d) ϕ

22. The function $f(x) = \cos\{\log_{10}(x + \sqrt{x^2 + 1})\}$, is

a) Even b) Odd c) Constant d) None of these

23. The period of the function $f(\theta) = 4 + 4\sin^3\theta - 3\sin\theta$ is
a) $\frac{2\pi}{3}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) π

24. If $f(2x+3) = \sin x + 2^x$, then $f(4m - 2n + 3)$ is equal to
a) $\sin(m-2m) + 2^{2m-n}$
b) $\sin(2m-n) + 2^{(m-n)2}$
c) $\sin(m-2n) + 2^{(m+n)2}$
d) $\sin(2m-n) + 2^{2m-n}$

25. The range of the function $f(x) = \frac{x+2}{x^2-8x-4}$, is
a) $\left(-\infty, \frac{-1}{4}\right] \cup \left[\frac{-1}{20}, \infty\right)$
b) $\left(-\infty, -\frac{1}{4}\right) \cup \left(-\frac{1}{20}, \infty\right)$
c) $\left(-\infty, -\frac{1}{4}\right] \cup \left(-\frac{1}{20}, \infty\right)$
d) None of these

26. Let $f: R \to R$ be a function defined by $f(x) = \cos(5x+2)$. Then, f is
a) Injective b) Surjective c) Bijective d) None of these

27. Which one is not periodic?
a) $|\sin 3x| + \sin^2 x$ b) $\cos\sqrt{x} + \cos^2 x$ c) $\cos 4x + \tan^2 x$ d) $\cos^2 x + \sin x$

28. If $f: R \to R$ is defined by $f(x) = [2x] - 2[x]$ for all $x \in R$, where $[x]$ is the greatest integer not exceeding x, then the range of f is
a) $\{x \in R: 0 \le x \le 1\}$ b) $\{0, 1\}$ c) $\{x \in R: x > 0\}$ d) $\{x \in R: x \le 0\}$

29. If $f(x) = \sin^2 x$ and the composite function $g(f(x)) = |\sin x|$, then the function $g(x)$ is equal to
a) $\sqrt{x-1}$ b) $\sqrt{x}$ c) $\sqrt{x+1}$ d) $-\sqrt{x}$

30. If a function $f: [2, \infty) \to B$ defined by $f(x) = x^2 - 4x + 5$ is a bijection, then $B =$
a) R b) $[1, \infty)$ c) $[4, \infty)$ d) $[5, \infty)$

31. The domain of definition of the function
$f(x) = \log_2[-(\log_2 x)^2 + 5\log_2 x - 6]$, is
a) $(4, 8)$ b) $[4, 8]$ c) $(0, 4) \cup (8, \infty)$ d) $R - [4, 8]$

32. The period of the function $f(x) = \sin\left(\sin\frac{x}{5}\right)$ is
a) 2π b) $2\pi/5$ c) 10π d) 5π

33. The domain of definition of the function

$$f(x) = \sqrt{\log_{10}\left(\frac{5x - x^2}{4}\right)}, \text{ is}$$

a) $[1, 4]$ b) $(1, 4)$ c) $(0, 5)$ d) $[0, 5]$

34. If $f: R \to R$ and is defined by $f(x) = \frac{1}{2-\cos 3x}$ for each $x \in R$, then the range f f is
a) $(1/3, 1)$ b) $[1/3, 1]$ c) $(1, 2)$ d) $[1, 2]$

35. If $f(x)$ is defined on $[0, 1]$ by the rule
$$f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 1 - x, & \text{if } x \text{ is irrational} \end{cases}$$
Then, for all $x \in [0, 1]$, $f(f(x))$ is
a) Constant b) $1 + x$ c) x d) None of these

36. Range of the function $f(x) = \frac{x}{1+x^2}$ is
a) $(-\infty, \infty)$ b) [-1, 1] c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ d) $[-\sqrt{2}, \sqrt{2}]$

37. If the function $f: R \to A$ given by $f(x) = \frac{x^2}{x^2+1}$ is a surjection, then $A =$
a) R b) $[0, 1]$ c) $(0, 1]$ d) $[0, 1)$

38. If R is an equivalence relation on a set A, then R^{-1} is
a) Reflexive only
b) Symmetric but not transitive
c) Equivalence
d) None of the above

39. If the function $f: R \to A$ given by $f(x) = \frac{x^2}{x^2+1}$ is a surjection, then $A =$
a) R
b) $[0, 1]$
c) $(0, 1]$
d) $[0, 1)$

40. The domain of the real valued function
$f(x) = \sqrt{5 - 4x - x^2} + x^2 \log(x + 4)$ is
a) $-5 \leq x \leq 1$
b) $-5 \leq x$ and $x \geq 1$
c) $-4 < x \leq 1$
d) ϕ

41. The period of the function $f(x) = a^{\{\tan(\pi x) + x - [x]\}}$, where $a > 0$, $[\cdot]$ denotes the greatest integer function and x is a real number, is
a) π
b) $\frac{\pi}{2}$
c) $\frac{\pi}{4}$
d) 1

42. The domain of the function $f(x) = \log_{2x-1}(x - 1)$ is
a) $(1, \infty)$
b) $\left(\frac{1}{2}, \infty\right)$
c) $(0, \infty)$
d) None of these

43. The composite mapping fog of the maps $f: R \to R, f(x) = \sin x$ and $g: R \to R, g(x) = x^2$, is
a) $x^2 \sin x$
b) $(\sin x)^2$
c) $\sin x^2$
d) $\frac{\sin x}{x^2}$

44. If $f(x) = \cos(\log x)$, then
$f(x)f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$ has the value
a) -1
b) 1/2
c) -2
d) 0

45. The domain of the function $f(x)$ given by
$f(x) = \sqrt{\frac{-\log_{0.3}(x-1)}{-x^2 + 3x + 18}}$, is
a) $[2, 6]$
b) $(2, 6)$
c) $[2, 6)$
d) None of these

46. If the function $f: R \to R$ defined by $f(x) = [x]$ where $[x]$ is the greatest integer not exceeding x, for $x \in R$, then f is
a) Even
b) Odd
c) Neither even nor odd
d) Strictly increasing

47. The domain of definition of the function
$f(x) = \log_3\left\{-\log_4\left(\frac{6x-4}{6x+5}\right)\right\}$, is
a) $(2/3, \infty)$
b) $(-\infty, -5/6) \cup (2/3, \infty)$
c) $[2/3, \infty)$
d) $(-5/6, 2/3)$

48. Which of the following statements is not correct for the relation R defined by aRb, if and only, if b lives within on kilometre from a?
a) R is reflexive
b) R is symmetric
c) R is anti-symmetric
d) None of these

49. Let $n(A) = 4$ and $n(B) = 6$. The number of one to one functions from A to B is
a) 24
b) 60
c) 120
d) 360

50. If $f(x) = x - \frac{1}{x}, x \neq 0$, then $f(x^2)$ equals
a) $f(x) + f(-x)$
b) $f(x)f(-x)$
c) $f(x) - f(-x)$
d) None of these

51. Let $f(x) = |x - 1|$ Then,
a) $f(x^2) = [f(x)]^2$
b) $f(|x|) = |f(x)|$
c) $f(x + y) = f(x) + f(y)$
d) None of these

52. If f is a real valued function such that $f(x + y) = f(x) + f(y)$ and $f(1) = 5$, then the value of $f(100)$ is

a) 200 b) 300 c) 350 d) 500

53. If R be a relation defined as aRb iff $|a-b|>0$, then the relation is
a) Reflexive
b) Symmetric
c) Transitive
d) Symmetric and transitive

54. Which of the following functions is inverse of itself?
a) $f(x)=\frac{1-x}{1+x}$
b) $f(x)=3^{\log x}$
c) $f(x)=3^{x(x+1)}$
d) None of these

55. The function $f(x)=\log(x+\sqrt{x^2+1})$ is
a) An even function
b) An odd function
c) A periodic function
d) Neither an even nor an odd function

56. If $b^2-4ac=0$ and $a>0$, then domian of the function $f(x)=\log\{(ax^2+bx+c)(x+1)\}$ is
a) $R-\left(-\frac{b}{2a}\right)$
b) $R-(-\infty,-1)$
c) $(-1,\infty)-\left\{-\frac{b}{2a}\right\}$
d) $R-\left(\left\{-\frac{b}{2a}\right\}\cap(-\infty,-1)\right)$

57. The function $f: R\to R$ given by $f(x)=x^2+x$, is
a) One-one and onto
b) One-one and into
c) Many-one and onto
d) Many one and into

58. If T_1 is the period of the function $f(x)=e^{3(x-[x])}$ and T_2 is the period of the function $g(x)=e^{3x-[3x]}$ ($[\cdot]$ denotes the greatest integer function), then
a) $T_1=T_2$
b) $T_1=\frac{T_2}{3}$
c) $T_1=3\,T_2$
d) None of these

59. If $f(x+y,x-y)=xy$, then the arithemetic mean of $f(x,y)$ and $f(y,x)$ is
a) x
b) y
c) 0
d) None of these

60. If $f: R\to R$ is defined by $f(x)=x-[x]-\frac{1}{2}$ for $x\in R$, where $[x]$ is the greatest integer not exceeding x, then
$\left\{x\in R: f(x)=\frac{1}{2}\right\}$ is equal to
a) Z, the set of all integers
b) N, the set of all natural numbers
c) ϕ, the empty set
d) R

61. The period of the function $f(x)=|\sin 3x|+|\cos 3x|$, is
a) $\frac{\pi}{2}$
b) $\frac{\pi}{6}$
c) $\frac{3\,\pi}{2}$
d) π

62. Let $f:(2,3)\to(0,1)$ be defined by $f(x)=x-[x]$, then $f^{-1}(x)$ equals
a) $x-2$
b) $x+1$
c) $x-1$
d) $x+2$

63. The function $f(x)=\left(\frac{1}{2}\right)^{\sin x}$, is
a) Periodic with period 2π
b) An odd function
c) Not expressible as the sum of an even function an odd function
d) None of these

64. If the function $f: N\to N$ is defined by $f(x)=\sqrt{x}$, then $\frac{f(25)}{f(16)+f(1)}$ is equal to
a) $\frac{5}{6}$
b) $\frac{5}{7}$
c) $\frac{5}{3}$
d) 1

65. Let $f: A\to B$ and $g: B\to C$ be two functions such that $gof: A\to C$ is onto. Then,
a) f is onto
b) g is onto
c) f and g both are onto
d) None of these

66. Let the function $f(x)=3x^2-4x+5\log(1+|x|)$ be defined on the interval $[0,1]$. The even extension of $f(x)$ to the interval $[-1,1]$ is
a) $3x^2+4x+8\log(1+|x|)$
b) $3x^2-4x+8\log(1+|x|)$
c) $3x^2+4x-8\log(1+|x|)$
d) None of these

67. Range of the function $f(x)=\frac{x^2+x+2}{x^2+x+1}; x \in R$ is

a) $(1,\infty)$ b) $(1, 11/7)$ c) $[1, 7/3]$ d) $(1, 7/5)$

68. The period of the function $\sin\left(\frac{\pi x}{2}\right)+\cos\left(\frac{\pi x}{2}\right)$ is

a) 4 b) 6 c) 12 d) 24

69. The period of the function $f(x)=\sin^4 x+\cos^4 x$ is

a) π b) $\pi/2$ c) 2π d) None of these

70. Let a relation R on the set N of natural numbers be defined as $(x,y) \Leftrightarrow x^2-4xy+3y^2=0 \forall x,y \in N$. The relation R is

a) Reflexive b) Symmetric

c) Transitive d) An equivalence relation

71. The function $f: R \to R$ defined by $f(x)=(x-1)(x-2)(x-3)$, is

a) One-one but not onto

b) Onto but not one-one

c) Both one and onto

d) Neither one-one nor onto

72. The function $f: X \to Y$ defined by $f(x)=\sin x$ is one-one but not onto, if X and Y are respectively equal to

a) R and R b) $[0,\pi]$ and $[0,1]$ c) $\left[0,\frac{\pi}{2}\right]$ and $[-1,1]$ d) $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ and $[-1,1]$

73. The function $f: R \to R$ is defined by $f(x)=3^{-x}$. Observe the following statements

I. f is one-one II. f is onto

III. f is a decreasing function

Out of these, true statement are

a) Only I, II b) Only II, III c) Only I, III d) I, II, III

74. The function $f(x)=x[x]$, is

a) Periodic with period 1

b) Periodic with period 2

c) Periodic with indeterminate period

d) Not-periodic

75. If $f(x)=\frac{3x+2}{5x-3}$, then

a) $f^{-1}(x)=f(x)$ b) $f^{-1}(x)=-f(x)$ c) $(fof)(x)=-x$ d) $f^{-1}(x)=-\frac{1}{19}f(x)$

76. The domain of the function $f(x)=\frac{\sqrt{4-x^2}}{\sin^{-1}(2-x)}$ is

a) $[0,2]$ b) $[0,2)$ c) $[1,2)$ d) $[1,2]$

77. The domain of definition of $f(x)=\sin^{-1}(|x-1|-2)$ is

a) $[-2,0]\cup[2,4]$ b) $(-2,0)\cup(2,4)$ c) $[-2,0]\cup[1,3]$ d) $[-2,0]\cup[1,3]$

78. The domain of the function $f(x)=\cos^{-1}[\sec x]$, where $[x]$ denotes the greatest integer less than or equal to x, is

a) $\{x : x=(2n+1)\pi, n \in Z\} \cup \left\{x : 2m\pi \le x < 2m\pi+\frac{\pi}{3}, m \in Z\right\}$

b) $\{x : x=2n\pi, n \in Z\} \cup \left\{x : 2m\pi < x < 2m\pi+\frac{\pi}{3}, m \in Z\right\}$

c) $\{x : (2n+1)\pi, n \in Z\} \cup \left\{x : 2m\pi < x < 2m\pi+\frac{\pi}{3}, m \in Z\right\}$

d) None of these

79. The domain of $\sin^{-1}(\log_3 x)$ is

a) [-1, 1] b) [0, 1] c) $[0,\infty]$ d) $\left[\frac{1}{3},3\right]$

80. Let $f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2}, (x \ne 0)$ then $f(x)$ equals

a) x^2-x for all x b) x^2-2 for all $|x| \ge 2$ c) x^2-2 for all $|x|<2$ d) None of these

81. If $f(x)=\sin^2 x+\sin^2\left(x+\frac{\pi}{3}\right)+\cos x\cos\left(x+\frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right)=1$, then $gof(x)$ is equal to

a) 1 b) -1 c) 2 d) -2

82. The range of $f(x)=\sec\left(\frac{\pi}{4}\cos^2 x\right), -\infty<x<\infty$, is

a) $[1,\sqrt{2}]$ b) $[1,\infty)$ c) $[-\sqrt{2},-1]\cup[1,\sqrt{2}]$ d) $(-\infty,-1]\cup[1,\infty)$

83. Let $f:R\to R$ be a function defined by $f(x)=\frac{e^{|x|}-e^{-x}}{e^x+e^{-x}}$. Then,

a) f is a bijection

b) f is an injection only

c) f is surjection on only

d) f is neither an injection nor a surjection

84. The function $f:R\to R$ defined by $f(x)=(x-1)(x-2)(x-3)$ is

a) One-one but not onto b) Onto but not one-one

c) Both one-one and onto d) Neither one-one nor onto

85. Q function f from the set of natural numbers to integers defined by $f(n)=\begin{cases}\frac{n-1}{2}, \text{where } n \text{ is odd}\\ -\frac{n}{2}, \text{when } n \text{ is even}\end{cases}$ is

a) One-one but not onto b) Onto but not one-one

c) One-one and onto both d) Neither one-one nor onto

86. The function $f(x)=\sqrt{\cos(\sin x)}+\sin^{-1}\left(\frac{1+x^2}{2x}\right)$ is defined for

a) $x\in\{-1,1\}$ b) $x\in[-,1,1]$ c) $x\in R$ d) $x\in(-1,1)$

87. If $e^{f(x)}=\frac{10+x}{10-x}, x\in(-10,10)$ and $f(x)=kf\left(\frac{200x}{100+x^2}\right)$, then k is equal to

a) 0.5 b) 0.6 c) 0.7 d) 0.8

88. A mapping $f:N\to N$, where N is the set of natural numbers is defined as

$f(n)=\begin{cases}n^2, \text{for } n \text{ odd}\\ 2n+1, \text{for } n \text{ even}\end{cases}$

For $n\in N$. Then, f is

a) Surjective but not injective b) Injective but not surjective

c) Bijective d) Neither injective nor surjective

89. If $y=f(x)=\frac{x+2}{x-1}$, then

a) $x=f(y)$ b) $f(1)=3$

c) y increase with x for $x<1$ d) f is a rational function of x

90. Let f be a function with domain $[-3,5]$ and let $g(x)=|3x+4|$. Then the domain of $(fog)(x)$ is

a) $\left(-3,\frac{1}{3}\right)$ b) $\left[-3,\frac{1}{3}\right]$ c) $\left[-3,\frac{1}{3}\right]$ d) $\left[-3,-\frac{1}{3}\right]$

91. Let $f:A\to B$ and g: $B\to C$ be two functions such that g$of:A\to C$ is one-one and $f:A\to B$ is onto. Then, g: $B\to C$ is

a) One-one b) Onto c) One-one and onto d) None of these

92. Let $g(x)=1+x-[x]$ and $f(x)=\begin{cases}-1, & x<0\\ 0, & x=0,\\ 1 & x>0\end{cases}$ then for for $x, f[g(x)]$ is equal to

a) x b) 1 c) $f(x)$ d) $g(x)$

93. If the function $f:R\to R$ be such that $f(x)=x-[x]$, where $[x]$ denotes the greatest integer less than or equal to x, then $f^{-1}(x)$, is

a) $\frac{1}{x-[x]}$ b) $[x]-x$ c) Not defined d) None of these

94. Let a and b be two integers such that $10a+b=5$ and $P(x)=x+ax+b$. The integer n such that $P(10).P(11)=P(n)$ is

a) 15 b) 65 c) 115 d) 165

95. The unction $f: [-1/2, 1/2] \to [-\pi/2, \pi/2]$ defined by $f(x) = \sin^{-1}(3x - 4x^3)$ is
a) Bijection
b) Injection but not a surjection
c) Surjection but not an injection
d) Neither an injection nor a surjection

96. Let $f: (-\infty, 2] \to (-\infty, 4]$ be a function defined by $f(x) = 4x - x^2$. Then, $f^{-1}(x)$ is
a) $2 - \sqrt{4 - x}$ b) $2 + \sqrt{4 - x}$ c) $2 \pm \sqrt{4 - x}$ d) Not defined

97. If $f(x) = (a - x^n)^{1/n}$, where $a > 0$ and $n \in N$, then $fof(x)$ is equal to
a) a b) x c) x^n d) a^n

98. The domain of definition of $f(x) = \log_3 |\log_e x|$, is
a) $(1, \infty)$ b) $(0, \infty)$ c) (e, ∞) d) None of these

99. Let $f: R \to, g: R \to R$ be two functions given by $f(x) = 2x - 3, g(x) = x^3 + 5$. Then, $(fog)^{-1}(x)$ is equal to
a) $\left(\frac{x+7}{2}\right)^{1/3}$ b) $\left(x - \frac{7}{2}\right)^{1/3}$ c) $\left(\frac{x-2}{7}\right)^{1/3}$ d) $\left(\frac{x-7}{2}\right)^{1}$

100. If $f: R \to R$ is defined by $f(x) = |x|$, then
a) $f^{-1}(x) = -x$
b) $f^{-1}(x) = \frac{1}{|x|}$
c) The function $f^{-1}(x)$ does not exist
d) $f^{-1}(x) = \frac{1}{x}$

101. Which of the following functions from $A = \{x : -1 \le x \le 1\}$ to itself are bijections?
a) $f(x) = \frac{x}{2}$ b) $g(x) = \sin\left(\frac{\pi x}{2}\right)$ c) $h(x) = |x|$ d) $k(x) = x^2$

102. Domain of the function $f(x) = \sqrt{2 - 2x - x^2}$ is
a) $-\sqrt{3} \le x \le +\sqrt{3}$
b) $-1 - \sqrt{3} \le x \le -1 + \sqrt{3}$
c) $-2 \le x \le 2$
d) $-2 + \sqrt{3} \le x \le -2 - \sqrt{3}$

103. Let $[x]$ denote the greatest integer less than or equal to x. If $f(x) = \sin^{-1} x, g(x) = [x^2]$ and $h(x) = 2x, \frac{1}{2} \le x \le \frac{1}{\sqrt{2}}$, then
a) $fogoh\,(x) = \pi/2$ b) $fogoh(x) = \pi$ c) $hofog = hogof$ d) $hofog \ne hogof$

104. Let $f : N \to N$ be defined by $f(x) = x^2 + x + 1$, then f is
a) One-one onto b) Many one onto c) One-one but not onto d) None of these

105. Let $f(x) = \begin{cases} 0, & x = 0 \\ x^2 \sin \pi/2x, & |x| < 1 \\ x|x|, & |x| \ge 1 \end{cases}$. Then, $f(x)$ is
a) An even function
b) An odd function
c) Neither an even function nor an odd function
d) $f'(x)$ is an even function

106. The interval in which the function $y = \frac{x-1}{x^2 - 3x + 3}$ transforms the real line is
a) $(0, \infty)$ b) $(-\infty, \infty)$ c) $[0, 1]$ d) $[-1/3, 1]$

107. The equivalent definition of
$f(x) = \max.\{x^2, (1-x)^2, 2x(1-x)\}$, where $0 \le x \le 1$,
a) $f(x) = \begin{cases} x^2; & 0 \le x \le 1/3 \\ 2x(1-x); & 1/3 \le x \le 2/3 \\ (1-x)^2; & 2/3 \le x \le 1 \end{cases}$
b) $f(x) = \begin{cases} (1-x)^2; & 0 \le x \le 1/3 \\ 2x(1-x); & 1/3 \le x \le 2/3 \\ x^2; & 2/3 \le x \le 1 \end{cases}$

c) $f(x)=\begin{cases} x^2 \ ; 0 \le x \le 1/2 \\ (1-x)^2 ; \ 1/2 \le x \le 1 \end{cases}$

d) None of these

108. Which of the following functions from Z to itself are bijections?

a) $f(x)=x^3$ b) $f(x)=x+2$ c) $f(x)=2x+1$ d) $f(x)=x^2+x$

109. The domain of definition of the function

$f(x)=\dfrac{1}{\sqrt{|\cos x|+\cos x}}$, is

a) $[-2n\pi, 2n\pi], n \in N$

b) $(2n\pi, (2n+1)\pi), n \in Z$

c) $\left((4n+1)\dfrac{\pi}{2}, (4n+3)\dfrac{\pi}{2}\right), n \in Z$

d) $\left((4n-1)\dfrac{\pi}{2}, (4n+1)\dfrac{\pi}{2}\right), n \in Z$

110. If $f(x)=(25-x^4)^{1/4}$ for $0<x<\sqrt{5}$, then $\left(f\left(\frac{1}{2}\right)\right)=$

a) 2^{-4} b) 2^{-3} c) 2^{-2} d) 2^{-1}

111. The function $f(x)=\sec[\log(x+\sqrt{1+x^2})]$ is

a) Odd b) Even c) Neither odd nor even d) Constant

112. If $f(x)=\sin(\log x)$, then the value of $f(xy)+f(x/y)-2f(x)\cos(\log y)$, is

a) -1 b) 0 c) 1 d) None of these

113. The equivalent definition of

$f(x)=\max.\left\{-|1-x^2|, 2|x|-2, 1-\dfrac{7}{2}|x|\right\}$, is

a) $\begin{cases} -2x+2 \ , \ x-1 \\ x^2-1 \ , \quad -1 \le x < 1/2 \\ 1+7x/2 \ , -1/2 \le x < 0 \\ 1-7x/2 \ , \quad 0 \le x < 1/2 \\ x^2-1 \ , \quad 1/2 \le x < 1 \\ 2x-2 \ , \quad x \ge 1 \end{cases}$

b) $\begin{cases} -2x-2 \ , \ x<-1 \\ -x^2-1 \ , \quad -1 \le x < \dfrac{1}{2} \\ 1+7x/2, \quad -1/2 \le x < 0 \\ 1-7x/2, \ 0 \le x < 1/2 \\ x^2-1 \ , \ 1/2 \le x < 1 \\ 2x-2 \ , \ x \ge 1 \end{cases}$

c) $\begin{cases} -2x+2 \ , \quad x \le -1 \\ x^2-1 \ , -1 \le x < 0 \\ 1+7x \ , \ 0 \le x < 1 \\ 2x-2 \ , \ x \ge 1 \end{cases}$

d) None of these

114. The number of bijective functions from set A to itself when A contains 106 elements is

a) 106 b) $(106)^2$ c) $106!$ d) 2^{106}

115. The domain of definition of

$f(x)=\log_{0.5}\left\{-\log_2\left(\dfrac{3x-1}{3x+2}\right)\right\}$, is

a) $(-\infty, -1/3)$ b) $(-1/3, \infty)$ c) $(1/3, \infty)$ d) $[1/3, \infty)$

116. If $f(x)=x^3-x$ and $\phi(x)=\sin 2x$, then

a) $\phi(f(2))=\sin 2$ b) $\phi(f(1))=1$ c) $f(\phi(\pi/12))=-\dfrac{3}{8}$ d) $f(f(1))=2$

117. $f(x)=|\sin x|$ has an inverse if its domain is

a) $[0, \pi]$ b) $[0, \pi/2]$ c) $[-\pi/4, \pi/4]$ d) None of these

118. The function $f(x) = \log_{10}(x + \sqrt{x^2+1})$ is

a) An even function b) An odd function c) Periodic function d) None of these

119. Let R be a relation on the set of integers given by $aRb \Leftrightarrow a = 2^k.b$ for some integer k. Then, R is

a) An equivalence relation
b) reflexive but not symmetric
c) Reflexive and transitive but not symmetric
d) Reflexive and symmetric but not transitive

120. A polynomial function $f(x)$ satisfies the condition

$f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$

If $f(10) = 1001$, then $f(20) =$

a) 2002 b) 8008 c) 8001 d) None of these

121. The function $f(x) = \frac{\sin^4 x + \cos^4 x}{x^3 + x^4 \tan x}$ is

a) Even
b) Odd
c) Periodic with period π
d) Periodic with period 2π

122. The value of b and c for which the identify $f(x+1) - f(x) = 8x + 3$ is satisfied, where $f(x) = bx^2 + cx + d$, are

a) $b = 2, c = 1$ b) $b = 4, c = -1$ c) $b = -1, c = 4$ d) $b = -1, c = 1$

123. The second degree polynomial $f(x)$, satisfying $f(0) = 0, f(1) = 1, f'(x) > 0$ for all $x \in (0, 1)$

a) $f(x) = \phi$
b) $f(x) = ax + (1-a)x^2; \forall a \in (0, \infty)$
c) $f(x) = ax + (1-a)x^2, a \in (0, 2)$
d) No such polynomial

124. If $2f(x+1) + f\left(\frac{1}{x+1}\right) = 2x$ and $x \neq -1$, then $f(2)$ is equal to

a) -1 b) 2 c) 5/3 d) 5/2

125. $f(x) = \begin{cases} x, \text{if } x \text{ is rational} \\ 0, \text{if } x \text{ is irrational} \end{cases}$ and

$f(x) = \begin{cases} 0, \text{if } x \text{ is rational} \\ x, \text{if } x \text{ is irrational} \end{cases}$. Then, $f - g$ is

a) One-one and into
b) Neither one-one nor onto
c) Many one and onto
d) One-one and onto

126. The value of x for which $y = \log_2\left\{-\log_{1/2}\left(1 + \frac{1}{x^{1/4}}\right) - 1\right\}$ is a real number are

a) $[0, 1]$ b) $(0, 1)$ c) $[1, \infty)$ d) None of these

127. If $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + [\log_{10}(3-x)]^{-1}$, then its domain is

a) $[-2, 6]$ b) $[-6, 2) \cup (2, 3)$ c) $[-6, 2]$ d) $[-2, 2) \cup (2,3]$

128. The range of the function

$f(x) = 1 + \sin x + \sin^3 x + \sin^5 x + \cdots$ when $x \in (-\pi/2, \pi/2)$, is

a) $(0, 1)$ b) R c) $(-2, 2)$ d) None of these

129. The number of onto mappings from the set $A = \{1, 2, \dots, 100\}$ to set $B = \{1, 2\}$ is

a) $2^{100} - 2$ b) 2^{100} c) $2^{99} - 2$ d) 2^{99}

130. If a function f satisfies $f\{f(x)\} = x + 1$ for all real values of x and if $f(0) = \frac{1}{2}$, then $f(1)$ is equal to

a) $\frac{1}{2}$ b) 1 c) $\frac{3}{2}$ d) 2

131. The function $f(x)$ given by

$f(x) = \frac{\sin 8x \cos x - \sin 6x \cos 3x}{\cos x \cos 2x - \sin 3x \sin 4x}$, is

a) Periodic with period π
b) Periodic with period 2π
c) Periodic with period $\pi/2$

d) Not periodic

132. If $x \in R$, then $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is equal to

a) $2\tan^{-1}x$

b) $\begin{cases} -\pi - 2\tan^{-1}x, -\infty < x < -1 \\ 2\tan^{-1}x, \quad -1 \le x \le 1 \\ \pi - 2\tan^{-1}x, 1 < x < \infty \end{cases}$

c) $\begin{cases} -\pi - 2\tan^{-1}x, -\infty < x < -1 \\ 2\tan^{-1}x, -1 \le x \le 1 \\ \pi - 2\tan^{-1}x, \ 1 < x < \infty \end{cases}$

d) $\begin{cases} -\pi + 2\tan^{-1}x, -\infty < x \le -1 \\ 2\tan^{-1}x, -1 < x < 1 \\ \pi - 2\tan^{-1}x, \ 1 \le x < \infty \end{cases}$

133. If $f(x) = 2x^6 + 3x^4 + 4x^2$, then $f'(x)$ is

a) An even function b) An odd function c) Neither even nor odd d) None of the above

134. The mapping $f: N \to N$ given $f(n) = 1 + n^2, n \in N$ where N is the set of natural number, is

a) One-to-one and onto b) Onto but not one-to-one

c) One-to-one but not onto d) Neither one-to-one nor onto

135. Let $f: A \to B$ and $g: B \to A$ be two functions such that $gof = I_A$. Then,

a) f is an injection and g is a surection

b) f is a surjection and g is an injection

c) f and g both are injections

d) f and g both are surjections

136. If $f(x) = (a - x^n)^{1/n}$, where $a > 0$ and $n \in N$, then $fof(x)$ is equal to

a) a b) x c) x^n d) a^n

137. Let r be a relation from R (set of real numbers) to R defined by $r = \{(a,b) | a, b \in R$ and $a - b + \sqrt{3}$ is an irrational number$\}$. The relation r is

a) An equivalent relation b) Reflexive only

c) Symmetric only d) Transitive only

138. R is a relation from {11, 12, 13} to {8, 10, 12} defined by $y = x - 3$. Then, R^{-1} is

a) {(8, 11), (10, 13)} b) {(11, 18), (13, 10)} c) {(10, 13), (8, 11)} d) None of these

139. If $f: R \to R$ is defined by $f(x) = x^2 - 6x - 14$, then $f^{-1}(2)$ equals to

a) {2, 8} b) {-2, 8} c) {-2, -8} d) {ϕ}

140. The domain of definition of the function

$f(x) = \sqrt[3]{\dfrac{2x+1}{x^2 - 10x - 11}}$, is

a) $(0, \infty)$ b) $(-\infty, 0)$ c) $R - \{-1, 11\}$ d) R

141. The period of the function $\sin\left(\frac{2x}{3}\right) + \sin\left(\frac{3x}{2}\right)$ is

a) 2π b) 10π c) 6π d) 12π

142. The function $f(x)$ which satisfies $f(x) = f(-x) = \frac{f'(x)}{x}$, is given by

a) $f(x) = \dfrac{1}{2}e^{x^2}$ b) $f(x) = \dfrac{1}{2}e^{-x^2}$ c) $f(x) = x^2 e^{x^2/2}$ d) $f(x) = e^{x^2/2}$

143. On the set of integers Z, define $f: Z \to Z$ as $f(n) = \begin{Bmatrix} \frac{n}{2}, n \text{ is even} \\ 0, \ n \text{ is odd} \end{Bmatrix}$, then '$f$'is

a) Injective but not surjective b) Neither injective nor surjective

c) Surjective but not injective d) Bijective

144. The maximum possible domain D and the corresponding range E, for the real function $f(x) = (-1)^x$ to exist is

a) $D = R, E = [-1, 1]$

b) $D = I$ (the set of integers), $E = [-1,1]$

c) $D = R, E = (-1,1)$

d) $D = I, E = \begin{cases} +1 \text{ when } x = 0 \text{ or even} \\ -1, \text{ when } x \text{ is odd} \end{cases}$

145. If $f: R \to R$, defined by $f(x) = x^2 + 1$, then the values of $f^{-1}(17)$ and $f^{-1}(-3)$ respectively are

a) $\phi, \{4, -4\}$ b) $\{3, -3\}, \phi$ c) $\{4, -4\}, \phi$ d) $\{4, -4\}, \{2, -2\}$

146. Let $f: A \to B$ and $g: B \to C$ be two functions such that $gof: A \to C$ is one-one. Then,

a) f is one-one b) f is one-one c) f is both are one-one d) None of these

147. Let $A = \{x \in R: x \neq 0, -4 \leq x \leq 4\}$ and $f: A \in R$ be defined by $f(x) = \frac{|x|}{x}$ for $x \in A$. Then, the range of f is

a) $\{1, -1\}$ b) $\{x: 0 \leq x \leq 4\}$ c) $\{1\}$ d) $\{x: -4 \leq x \leq 0\}$

148. If $f(x) = (9x + 0.5) \log_{(0.5+x)} \left(\frac{x^2+2x-3}{4x^2-4x-3}\right)$ is a real number, then x belongs to

a) $(-1/2, 1)$

b) $(-1/2, 1/2) \cup (1/2, 1) \cup (3/2, \infty)$

c) $(-1/2 - 1)$

d) None of these

149. Let the function f, g, h are defined from the set of real numbers R to R such that $f(x) = x^2 - 1, g(x) = \sqrt{(x^2 + 1)}$ and $h(x) = \begin{cases} 0, \text{if } x < 0 \\ x, \text{if } x \geq 0 \end{cases}$, then $ho(fog)(x)$ is defined by

a) x b) x^2 c) 0 d) None of these

150. The number of reflexive relations of a set with four elements is equal to

a) 2^{16} b) 2^{12} c) 2^8 d) 2^4

151. Let $f(x) = (ax^2 + b)^3$, then the function g satisfying $f(g(x)) = g(f(x))$ is given by

a) $g(x) = \left(\frac{b - x^{1/3}}{a}\right)^{1/2}$ b) $g(x) = \frac{1}{(a\,x^2 + b)^3}$ c) $g(x) = (a\,x^2 + b)^{1/3}$ d) $g(x) = \left(\frac{x^{1/3} - b}{a}\right)^{1/2}$

152. If $f(x) = \big||x| - 1\big|$, then $fof(x)$ equals

a) $f(x) = \begin{cases} |x| - 2, & |x| \geq 2 \\ 2 - |x|, & 1 < |x| < 2 \\ |x|, & |x| \leq 1 \end{cases}$

b) $f(x) = \begin{cases} |x| + 2, & |x| \geq 2 \\ |x| - 2, & 1 \leq |x| \leq 2 \\ |x|, & |x| \leq 2 \end{cases}$

c) $f(x) = \begin{cases} |x| - 2, & |x| \geq 2 \\ 2 + |x|, & 1 \leq |x| \leq 2 \\ |x|, & |x| \leq 1 \end{cases}$

d) None of these

153. The domain of definition of the function $f(x) = \tan\left(\frac{\pi}{[x+2]}\right)$, is

a) $[-2, 1]$ b) $(-2, -1)$ c) $R - [-2, -1)$ d) None of these

154. A function $f: A \to B$, where $A = \{x: -1 \leq x \leq 1\}$ and $B = \{y: 1 \leq y \leq 2\}$, is defined by the rule $y = f(x) = 1 + x^2$. Which of the following statement is true?

a) f is injective but not surjective b) f is surjective but not injective

c) f is both injective and surjective d) f is neither injective nor surjective

155. The function $f: R \to R$, defined by $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x, is

a) One-one

b) Onto

c) One-one and onto

d) Neither one-one nor onto

156. Let $f: A \to B$ and $g : B \to C$ be bijections, then $(fog)^{-1} =$

a) $f^{-1}og^{-1}$ b) fog c) $g^{-1}\,of^{-1}$ d) gof

157. Let $f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2}, x\neq 0$, then $f(x)$ is equal to

a) x^2 b) x^2-1 c) x^2-2 d) x^2+1

158. The relation $R=\{(1,1),(2,2),(3,3)\}$ on the set $\{1,2,3\}$ is

a) Symmetric only b) Reflexive only

c) An equivalence relation d) Transitive only

159. If $f(x)=a\,x+b$ and $g(x)=c\,x+d$, then $f\big(g(x)\big)=g\big(f(x)\big)\Leftrightarrow$

a) $f(a)=g(c)$ b) $f(b)=g(b)$ c) $f(d)=g(b)$ d) $f(c)=g(a)$

160. If $f:R\rightarrow R$ is defined by $f(x)=2x-2[x]$ for all $x\in R$, where $[x]$ denotes the greatest integer less than or equal to x, then range of f, is

a) $[0,1]$ b) $\{0,1\}$ c) $(0,\infty)$ d) $(-\infty,0]$

161. The domain of definition of $f(x)=\log_{10}\{\log_{10}(1+x^3)\}$, is

a) $(-1,\infty)$ b) $(0,\infty)$ c) $[0,\infty)$ d) $(-1,0)$

162. Let $R=\{(3,3),(6,6),(9,9),(12,12),(6,12),(3,9),(3,12),(3,6)\}$ be a relation on the set $A=\{3,6,9,12\}$. The relation is

a) Reflexive and symmetric only b) An equivalence relation

c) Reflexive only d) Reflexive and transitive only

163. If $f(x)=a^x$, which of the following equalities hold?

a) $f(x+2)-2f(x+1)+f(x)=(a-1)^2f(x)$

b) $f(-x)f(x)+1=0$

c) $f(x+y)=f(x)+f(y)$

d) $f(x+3)-2f(x+2)+f(x+1)=(a-2)^2f(x+1)$

164. The inverse of the function $f(x)=\frac{10^x-10^{-x}}{10^x+10^{-x}}+1$ is given by

a) $\frac{1}{2}\log_{10}\left(\frac{x}{2-x}\right)$ b) $\log_{10}\left(\frac{x}{2-x}\right)$ c) $\frac{1}{2}\log_{10}\left(\frac{x}{1-x}\right)$ d) None of these

165. If $f(x)=\sqrt{|3^x-3^{1-x}|-2}$ and $g(x)=\tan\pi\,x$, then domain of $fog\;(x)$ is

a) $\left[n+\frac{1}{3},n+\frac{1}{2}\right]\cup\left[n+\frac{1}{2},n+1\right], n\in Z$

b) $\left(nx+\frac{1}{4},n+\frac{1}{2}\right)\cup\left(n+\frac{1}{2},n+1\right), n\in Z$

c) $\left(n+\frac{1}{4},n+\frac{1}{2}\right)\cup\left[n-\frac{1}{2},n+1\right], n\in Z$

d) $\left[n+\frac{1}{4},x+\frac{1}{2}\right)\cup\left(n+\frac{1}{2},n+2\right), n\in Z$

166. If the functions f and g are defined by $f(x)=3x-4, g(x)=3x+2$ for $x\in R$, respectively then $g^{-1}(f^{-1}(5))=$

a) 1 b) 1/2 c) 1/3 d) 1/4

167. If $f(x)$ and $g(x)$ are two real functions such that $f(x)+g(x)=e^x$ and $f(x)-g(x)=e^{-x}$, then

a) $f(x)$ is an odd function

b) $g(x)$ is an even function

c) $f(x)$ and $g(x)$ are periodic functions

d) None of these

168. Let $f(x)=\frac{1}{2}-\tan\left(\frac{\pi x}{2}\right), -1<x<1$ and $g(x)=\sqrt{3+4x-4x^2}$, then dom $(f+g)$ is given by

a) $\left[\frac{1}{2},1\right]$ b) $\left[\frac{1}{2},-1\right)$ c) $\left[-\frac{1}{2},1\right)$ d) $\left[-\frac{1}{2},-1\right]$

169. If $f(x)=2x^6+3x^4+4x^2$, then $f'(x)$ is

a) Even function b) An odd function c) Neither even nor odd d) None of these

170. The domain of the function $f(x)=\sqrt{\cos^{-1}\left(\frac{1-|x|}{2}\right)}$ is

a) (-3, 3) b) [-3, 3] c) $(-\infty,-3)\cup(3,\infty)$ d) $(-\infty,-3]\cup[3,\infty)$

171. Which of the following functions is one-to -one?

a) $f(x)=\sin x, x\in[-\pi,\pi]$ b) $f(x)=\sin x, x\in\left[-\frac{3\pi}{2},-\frac{\pi}{4}\right]$

c) $f(x)=\cos x, x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ d) $f(x)=\cos x, x\in\left[\pi,\frac{3\pi}{2}\right]$

172. Given $f(x)=\log\left(\frac{1+x}{1-x}\right)$ and $g(x)=\frac{3x+x^3}{1+3x^2}$, then $fog(x)$ equals

a) $-f(x)$ b) $3f(x)$ c) $[f(x)]^3$ d) None of these

173. The largest possible set of real numbers which can be the domain of $f(x)=\sqrt{1-\frac{1}{x}}$ is

a) $(0,1)\cup(0,\infty)$ b) $(-1,0)\cup(1,\infty)$ c) $(-\infty,-1)\cup(0,\infty)$ d) $(-\infty,0)\cup[1,\infty)$

174. The set of values of a for which the function $f(x)=\sin x+\left[\frac{x^2}{a}\right]$ defined on $[-2,2]$ is an odd function, is

a) $(4,\infty)$ b) $[-4,4]$ c) $(-\infty,4)$ d) None of these

175. On the set N of all natural numbers define the relation R by aRbif and only if the GCD of a and b is 2, then R is

a) Reflexive, but not symmetric b) Symmetric only

c) Reflexive, and transitive d) Reflexive, symmetric and transitive

176. Let $f(x)$ be a real valued function defined by
$f(x+\lambda)=1+[2-5f(x)+10\{f(x)\}^2-10\{f(x)\}^3+5\{f(x)\}^4-\{f(x)\}^5]^{1/5}$ for all real x and some positive constant λ, then $f(x)$ is

a) A periodic function with period λ

b) A periodic function with period 2λ

c) Not a periodic function

d) A periodic function with indeterminate period

177. The domain of the function $f(x)=\sqrt{\log_{10}\left(\frac{1}{|\sin x|}\right)}$, is

a) $R-\{-\pi,\pi\}$ b) $R-\{n\pi|n\in Z\}$ c) $R-\{2n\pi|n\in z\}$ d) $(-\infty,\infty)$

178. The function $f(x)=\log\left(\frac{1+x}{1-x}\right)$ satisfies the equation

a) $f(x+2)-2f(x+1)+f(x)=0$ b) $f(x)+f(x+1)=f\{x(x+1)\}$

c) $f(x)+f(y)=f\left(\frac{x+y}{1+xy}\right)$ d) $f(x+y)=f(x)f(y)$

179. If $f(x)$ is defined on [0, 1], then the domain of definition of $f(\tan x)$ is

a) $[n\pi, n\pi+\pi/4], n\in Z$

b) $[2n\pi, 2n\pi+\pi/4], n\in Z$

c) $[n\pi-\pi/4, n\pi+\pi/4], n\in Z]$

d) None of these

180. If a function F is such that $F(0)=2, F(1)=3, F(n+2)=2F(n)-F(n+1)$ for $n\neq 0$, then $F(5)$ is equal to

a) -7 b) -3 c) 7 d) 13

181. $f(x)=\sqrt{\sin^{-1}(\log_2 x)}$ exists for

a) $x\in(1,2)$ b) $x\in[1,2]$ c) $x\in[2,\infty)$ d) $x\in(0,\infty)$

182. The function $f(x)=\begin{cases}1, & x\in Q\\ 0, & x\notin Q\end{cases}$ is

a) Periodic with period 1

b) Periodic with period 2

c) Not periodic

d) Periodic with indeterminate period

183. The function $f(x) = \frac{\sec^4 x + \text{cosec}^4 x}{x^3 + x^4 \cot x}$ is
a) Even
b) Odd
c) Neither even nor odd
d) Periodic with period π

184. The function $f(x) = |\cos x|$ is periodic with period
a) 2π b) π c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

185. If $f(x) = x^n, n \in N$ and $gof(x) = n\, g(x)$, then $g(x)$ can be
a) $n|x|$ b) $3x^{1/3}$ c) e^x d) $\log|x|$

186. If $f(x)$ is an odd function, then the curve $y = f(x)$ is symmetric
a) About x-axis
b) About y-axis
c) About both the axes
d) In opposite quadrants

187. If the function $f: [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is
a) $\left(\frac{1}{2}\right)^{x(x-1)}$ b) $\frac{1}{2}(1 + \sqrt{1 + 4\log_2 y})$
c) $\frac{1}{2}(1 - \sqrt{1 + 4\log_2 y})$ d) ∞

188. If $f: R \to R$ and $g: R \to R$ are defined by $f(x) = |x|$ and $g(x) = [x - 3]$ for $x \in R$, then $\left\{g(f(x)): -\frac{8}{5} < x < \frac{8}{5}\right\}$ is equal to
a) {0, 1} b) {1, 2} c) {-3, -2} d) {2, 3}

189. The domain of definition of
$f(x) = \log_{10}\{1 - \log_{10}(x^5 - 5x + 16)\}$, is
a) $(1, 3)$ b) $(2, 3)$ c) $[2, 3]$ d) None of these

190. The period of the function $f(x) = \sin^2 x + \cos^4 x$ is
a) π b) $\frac{\pi}{2}$ c) 2π d) None of these

191. If $f(x) = \sin x + \cos x$, $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain
a) $\left[0, \frac{\pi}{2}\right]$ b) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ d) $[0, \pi]$

192. Domain of definition of the function $f(x) = \sqrt{\sin^{-}(2x) + \frac{\pi}{6}}$ for real valued x, is
a) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ c) $\left(-\frac{1}{2}, \frac{1}{9}\right)$ d) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

193. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then $f\left(\frac{2x}{1+x^2}\right)$ will be equal to
a) $2f(x^2)$ b) $f(x^2)$ c) $2f(2x)$ d) $2f(x)$

194. The domain of $f(x) = \log|\log_e x|$, is
a) $(0, \infty)$ b) $(1, \infty)$ c) $(0,1) \cup (1, \infty)$ d) $(-\infty, 1)$

195. If $f(x)$ is an even function, then the curve $y = f(x)$ is symmetric about
a) x-axis b) y-axis c) Both the axes d) None of these

196. If $f(x) = \left(\frac{x}{1-|x|}\right)^{1/2002}$, then D_f is
a) $R - [-1, 1]$ b) $(-\infty, 1)$ c) $(-\infty, -1) \cup (0, 1)$ d) None of these

197. If $f(x) = \begin{cases} [x], \text{if } -3 < x \le -1 \\ |x|, \text{if } -1 < x < 1 \\ |[x]|, \text{if } 1 \le x < 3 \end{cases}$, then the set $(x: f(x) \ge 0)$to
a) (-1, 3) b) [-1, 3) c) (-1, 3] d) [-1, 3]

198. If $f(x)=\frac{x}{x-1}, x \neq 1$ then $\underbrace{(f o f o \ldots o f)(x)}_{19 \text{ times}}$ is equal to

a) $\frac{x}{x-1}$ b) $\left(\frac{x}{x-1}\right)^{19}$ c) $\frac{19x}{x-1}$ d) x

199. The domain of the function $f(x)=\log_{10}(\sqrt{x-4}+\sqrt{6-x})$, is

a) $[4,6]$ b) $(-\infty, 6)$ c) $(2,3)$ d) None of these

200. If $f: N \rightarrow N$ is defined by $f(n)=$ the sum of positive divisors of n, then $f(2^k \times 3)$, where k is a positive integer, is

a) $2^{k+1}-1$ b) $2(2^{k+1}-1)$ c) $3(2^{k+1}-1)$ d) $4(2^{k+1}-1)$

201. Let $A=\{x:-1 \leq x \leq 1\}$ and $f: A \rightarrow A$ such that $f(x)=x|x|$, then f is

a) A bijection

b) Injective but not surjective

c) Surjective but not injective

d) Neither injective nor surjective

202. The domain of the function $\sin^{-}\left(\log_2 \frac{x^2}{2}\right)$ is

a) [-1, 2]-{0} b) [-2, 2]-(-1, 1) c) [-2, 2]-{0} d) [1, 2]

203. If $f(x)=ax+b$ and $g(x)=cx+d$, then $f\{g(x)\}=g\{f(x)\}$ is equivalent to

a) $f(a)=f(c)$ b) $f(b)=g(b)$ c) $f(d)=g(b)$ d) $f(c)=g(a)$

204. The period of the function $f(x)=\sin^4 3x+\cos^4 3x$ is

a) $\pi/2$ b) $\pi/3$ c) $\pi/6$ d) None of these

205. Given $f(x)=\log_{10}\left(\frac{1+x}{1-x}\right)$ and $g(x)=\frac{3x+x^3}{1+3x^2}$, then $fog\ (x)$ equals

a) $-f(x)$ b) $3f(x)$ c) $[f(x)]^3$ d) None of these

206. Which of the following functions is not an are not an injective map(s)?

a) $f(x)=|x+1|, x \in[-1, \infty)$

b) $g(x)=x+\frac{1}{x}, x \in(0, \infty)$

c) $h(x)=x^2+4x-5, x \in(0, \infty)$

d) $h(x)=e^{-x}, x \in[0, \infty)$

207. If $f: R \rightarrow R$ and $g: R \rightarrow R$ are defined by $f(x)=x-[x]$ and $g(x)=[x]$ for $x \in R$, where $[x]$ is the greatest integer not exceeding x, then for every $x \in R, f(g(x))$ is equal to

a) x b) 0 c) $f(x)$ d) $g(x)$

208. The domain of definition of $f(x)=\sqrt{\frac{\log_{0.3}|x-2|}{|x|}}$, is

a) $[1,2) \cup(2,3]$ b) $[1,3]$ c) $R-(1,3]$ d) None of these

209. $f: R \rightarrow R$ given by $f(x)=5-3\sin x$, is

a) One-one b) Onto c) One-one and onto d) None of these

210. If $f(x+2y, x-2y)=xy$, then $f(x, y)$ equals

a) $\frac{x^2-y^2}{8}$ b) $\frac{x^2-y^2}{4}$ c) $\frac{x^2+y^2}{4}$ d) $\frac{x^2-y^2}{2}$

211. If $f: R \rightarrow R$ is defined as $f(x)=(1-x)^{1/3}$, then $f^{-1}(x)$ is

a) $(1-x)^{-1/3}$ b) $(1-x)^3$ c) $1-x^3$ d) $1-x^{1/3}$

212. If $f(x+2y, x, x-2y)=xy$, then $f(x, y)$ equals

a) $\frac{x^2-y^2}{8}$ b) $\frac{x^2-y^2}{4}$ c) $\frac{x^2+y^2}{4}$ d) $\frac{x^2-y^2}{2}$

213. Let $f:[4, \infty[\rightarrow[4, \infty[$ be defined by $f(x)=5^{x(x-4)}$ then $f^{-1}(x)$

a) $2-\sqrt{4+\log_5 x}$ b) $2+\sqrt{4+\log_5 x}$ c) $\left(\frac{1}{5}\right)^{x(x-4)}$ d) Not defined

214. If $f: [2, 3] \to R$ is defined by $f(x) = x^3 + 3x - 2$, then the range $f(x)$ is contained in the interval

a) [1, 12] b) [12, 34] c) [35, 50] d) [-12, 12]

215. The period of $\sin^2 \theta$, is

a) π^2 b) π c) 2π d) $\pi/2$

216. If $n \in N$, and the period of $\frac{\cos nx}{\sin\left(\frac{x}{n}\right)}$ is 4π, then n is equal to

a) 4 b) 3 c) 2 d) 1

217. Foe real x, let $f(x) = x^3 + 5x + 1$, then

a) f is one-one but not onto R

b) f is onto R but not one-one

c) f is one-one and onto R

d) f is neither one-one nor onto R

218. The range of the function $f(x) = \frac{1}{2-\cos 3x}$, is

a) $[-1/3, 0]$ b) R c) $[1/3, 1]$ d) None of these

219. Let $A = \{2, 3, 4, 5, \dots, 16, 17, 18\}$. Let be the equivalence relation on $A \times A$, cartesian product of A and A, defined by $(a, b) \approx (c, d)$ if $ad = bc$, then the number of ordered pairs of the equivalence class of $(3, 2)$ is

a) 4 b) 5 c) 6 d) 7

220. Let n be the natural number. Then, the range of the function $f(n) = 8 - n_{P_{n-4}}$, $4 \le n \le 6$, is

a) $\{1, 2, 3, 4\}$ b) $\{1, 2, 3, 4, 5, 6\}$ c) $\{1, 2, 3\}$ d) $\{1, 2, 3, 4, 5\}$

221. Let X and Y be subsets of R, the set of all real numbers. The function $f: X \to Y$ defined by $f(x) = x^2$ for $x \in X$ is one-one but not onto, if (Here, R^+ is the set of all positive real numbers)

a) $X = Y = R^+$ b) $X = R, Y = R^+$ c) $X = R^+, Y = R$ d) $X = Y = R$

222. If $f(x).f(1/x) = f(x) + f(1/x)$ and $f(4) = 65$, then $f(6)$ is

a) 65 b) 217 c) 215 d) 64

223. The graph of the function of $y = f(x)$ is symmetrical about the line $x = 2$, then

a) $f(x + 2) = f(x - 2)$ b) $f(2 + x) = f(2 - x)$ c) $f(x) = f(-x)$ d) $f(x) = -f(-x)$

224. If $f(x) = \begin{cases} -1; & x < 0 \\ 0; & x = 0 \\ 1; & x > 0 \end{cases}$ and $g(x) = x(1 - x^2)$, then

a) $fog\,(x) = \begin{cases} -1; & -1 < x < 0 \text{ or } x > 1 \\ 0; & x = 0, 1, -1 \\ 1; & 0 < x < 1 \end{cases}$

b) $fog\,(x) = \begin{cases} -1; & -1 < x < 0 \\ 0; & x = 0,1,-1 \\ 1; & 0 < x < 1 \end{cases}$

c) $fog\,(x) = \begin{cases} -1; & -1 < x < 0 \text{ or } x > 1 \\ 0; & x = 0,\ 1, -1 \\ 1; & 0 < x < 1 \text{ or } x < -1 \end{cases}$

d) $fog\,(x) = \begin{cases} 1; & -1 < x < 0 \text{ or } x > 1 \\ 0; & x = 0,1,-1 \\ 1; & 0 < x < 1 \text{ or } x < -1 \end{cases}$

225. $x_2 = xy$ is a relation which is

a) Symmetric

b) Reflexive and transitive

c) Transitive

d) None of these

226. The period of

$f(x) = \sin\left(\frac{\pi x}{n-1}\right) + \cos\left(\frac{\pi x}{n}\right), n \in Z,\ n > 2$, is

a) $2n\,\pi(n - 1)$ b) $4(n - 1)\,\pi$ c) $2n\,(n - 1)$ d) None of these

227. $f : [-4,0] \to R$ is given by $f(x) = e^x + \sin x$, its even extension to $[-4,4]$, is

a) $-e^{|x} - \sin|x|$ b) $e^{-|x|} - \sin|x|$ c) $e^{-|x|} + \sin|x|$ d) $-e^{-|x|+\sin|x|}$

228. Let $f: R \to R$ be a function defined by $f(x) = -\frac{|x|^3+|x|}{1+x^2}$, then the graph of $f(x)$ lies in the

a) I and II quadrants b) I and III quadrants c) II and III quadrants d) III and IV quadrants

229. The domain of the real valued function $f(x) = \sqrt{1 - 2x} + 2\ \sin^{-1}\left(\frac{3x-1}{2}\right)$ is

a) $\left[-\frac{1}{3}, 1\right]$ b) $\left[\frac{1}{2}, 1\right]$ c) $\left[-\frac{1}{2}, \frac{1}{3}\right]$ d) $\left[-\frac{1}{3}, \frac{1}{2}\right]$

230. The domain of function $f(x) = \log_{(x+3)}(x^2 - 1)$ is
a) $(-3, -1) \cup (1, \infty)$
b) $[-3, -1) \cup [1, \infty)$
c) $(-3, -2) \cup (-2, -1) \cup (1, \infty)$
d) $[-3, -2) \cup (-2, -1) \cup [1, \infty)$

231. The range of the function $f(x) = x^2 - 6x + 7$ is
a) $(-\infty, 0)$ b) $[-2, \infty)$ c) $(-\infty, \infty)$ d) $(-\infty, -2)$

232. The inverse of the function $f: R \to (-1,3)$ is given by $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} + 2$
a) $\log\left(\frac{x-1}{x+1}\right)^{-2}$ b) $\log\left(\frac{x-2}{x-1}\right)^{1/2}$ c) $\log\left(\frac{x}{2-x}\right)^{1/2}$ d) $\log\left(\frac{x-1}{3-x}\right)^{1/2}$

233. If $f(x) = \frac{4^x}{4^x + 2}$, then $f\left(\frac{1}{97}\right) + f\left(\frac{2}{97}\right) + ... + f\left(\frac{96}{97}\right)$ is equal to
a) 1 b) 48 c) -48 d) -1

234. The period of the function
$f(x) = \frac{\sin 8x \cos x - \sin 6x \cos 3x}{\cos 2x \cos x - \sin 3x \sin 4x}$ is
a) π b) 2π c) $\frac{\pi}{2}$ d) None of these

235. Let $f: R \to R: f(x) = x^2$ and $g: R \to R: g(x) = x + 5$, then gof is
a) $(x + 5)$ b) $(x + 5^2)$ c) $(x^2 + 5^2)$ d) $(x^2 + 5)$

236. The function $f(x) = \log_{2x-5}(x^2 - 3x - 10)$ is defined for all x belonging to
a) $[5, \infty)$ b) $(5, \infty)$ c) $(-\infty, +5)$ d) None of these

237. Range of the function $f(x) = \frac{x^2}{x^2+1}$ is
a) (-1, 0) b) (-1, 1) c) [0, 1) d) (1, 1)

238. Let $f(x) = |x - 1|$. Then,
a) $f(x^2) = [f(x)]^2$
b) $f(|x|) = |f(x)|$
c) $f(x + y) = f(x) + f(y)$
d) None of these

239. If $f(x) = a^x$, which of the following equalities do not hold?
a) $f(x + 2) - 2f(x + 1) + f(x) = (a - 1)^2 f(x)$
b) $f(-x)f(x) - 1 = 0$
c) $f(x + y) = f(x)f(y)$
d) $f(x + 3) - 2\, f(x + 2) + f\, (x + 1) = (a - 2)^2 f\, (x + 1)$

240. Let $A = \{x \in R: x \le 1\}$ and $f: A \to A$ be defined as $f(x) = x(2 - x)$. Then, $f^{-1}(x)$ is
a) $1 + \sqrt{1 - x}$ b) $1 - \sqrt{1 - x}$ c) $\sqrt{1 - x}$ d) $1 \pm \sqrt{1 - x}$

241. The function $f(x) = \sin\frac{\pi x}{2} + 2\cos\frac{\pi x}{3} - \tan\frac{\pi x}{4}$ is periodic with period
a) 6 b) 3 c) 4 d) 12

242. The equivalent definition of the function
$f(x) = \lim_{n\to\infty} \frac{x^n - x^{-n}}{x^n + x^{-n}}, x > 0$, is
a) $f(x) = \begin{cases} -1, & 0 < x \le 1 \\ 1, & x > 1 \end{cases}$
b) $f(x) = \begin{cases} -1, & 0 < x < 1 \\ 1, & x \ge 1 \end{cases}$
c) $f(x) = \begin{cases} -1, & 0 < x < 1 \\ 0, & x = 1 \\ 1, & x > 1 \end{cases}$
d) None of these

243. Let $R = \{(1,3),(4,2),(2,4),(2,3),(3,1)\}$ be a relation on the set $A = \{1,2,3,4\}$. The relation R is

a) A function b) Transitive c) Not symmetric d) Reflexive

244. The domain of the function

$f(x) = {}^{16-x}C_{2x-1} + {}^{20-3x}P_{4x-5}$, where the symbols have their usual meanings, is the set

a) $\{2,3\}$ b) $\{2,3,4\}$ c) $\{1,2,3,4\}$ d) $\{1,2,3,4,5\}$

245. If $f: R \to C$ is defined by $f(x) = e^{2ix}$ for $x \in R$, then f is (where C denotes the set of all complex numbers)

a) One-one b) Onto

c) One-one and onto d) Neither one-one nor onto

246. The domain of the function

$f(x) = \log_{10}(\sqrt{x-4} + \sqrt{6-x})$ is

a) $[4,6]$ b) $(-\infty,6)$ c) $[2,3)$ d) None of these

247. If $f(x) = \sin^2 x, g(x) = \sqrt{x}$ and $h(x) = \cos^{-1} x, 0 \le x \le 1$, then

a) $hogof = fogoh$ b) $gofoh = fohog$ c) $fohog = hogof$ d) None of these

248. If $f(x) = \frac{2^x+2^{-x}}{2}$, then $f(x+y)f(x-y)$ is equal to

a) $\frac{1}{2}\{f(2x)+f(2y)\}$ b) $\frac{1}{2}\{f(2x)-f(2y)\}$ c) $\frac{1}{4}\{f(2x)+f(2y)\}$ d) $\frac{1}{4}\{f(2x)-f(2y)\}$

249. The relation R defined on the set of natural numbers as $\{(a,b): a$ differs from b by $3\}$ is given by

a) $\{(1,4),(2,5),(3,6),...\}$ b) $\{(4,1),(5,2),(6,3),...\}$

c) $\{(1,3),(2,6),(3,9),...\}$ d) None of the above

250. The domain of the function $f(x) = \sin^{-1}(\log_3(x/3))$ is

a) $[1,9]$ b) $[-1,9]$ c) $[-9,1]$ d) $[-9,-1]$

251. The range of the function $f(x) = \sin\left\{\log_{10}\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right\}$, is

a) $[0,1]$ b) $(-1,0)$ c) $[-1,1]$ d) $(-1,1)$

252. Let $f(x) = \frac{ax+b}{cx+d}$. Then, $fof(x) = x$ provided that

a) $d = -a$ b) $d = a$ c) $a = b = c = d = 1$ d) $a = b = 1$

253. Let C denote the set of all complex numbers. The function $f : C \to C$ defined by $f(x) = \frac{ax+b}{cx+d}$ for $x \in C$, where $bd \neq 0$ reduces to a constant function if:

a) $a = c$ b) $b = d$ c) $ad = bc$ d) $ab = cd$

254. If $\sin\lambda x + \cos\lambda x$ and $|\sin x| + |\cos x|$ are periodic function with the same period, then $\lambda =$

a) 0 b) 1 c) 2 d) 4

255. The domain of definition of the real function $f(x) = \sqrt{\log_{12} x^2}$ of the real variable x, is

a) $x > 0$ b) $|x| \ge 1$ c) $|x| \ge 4$ d) $x \ge 4$

256. If $f(x)$ is an even function and $f'(x)$ exists, then $f'(e) + f'(-e)$ is

a) >0 b) $= 0$ c) ≥ 0 d) <0

257. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to

a) $\{f(x)\}^2$ b) $\{f(x)\}^3$ c) $2f(x)$ d) $3f(x)$

258. If the function $f: R \to R$ is defined by $f(x) = \cos^2 x + \sin^4 x$ then $f(R) =$

a) $[3/4,1)$ b) $(3/4,1]$ c) $[3/4,1]$ d) $(3/4,1)$

259. The domain of $\sin^{-1}\left[\log_2\left(\frac{x}{12}\right)\right]$ is

a) $[2, 12]$ b) $[-1, 1]$ c) $\left[\frac{1}{3}, 24\right]$ d) $[6, 24]$

260. The largest interval lying in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which the function $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$ is defined, is

a) $[0,\pi]$ b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ c) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ d) $\left[0, \frac{\pi}{2}\right]$

261. Let $f: R \to R$ be define by $f(x) = 3x - 4$. Then, $f^{-1}(x)$ is

a) $\frac{x+4}{3}$ b) $\frac{x}{3}-4$ c) $3x+4$ d) None of these

262. The interval in which the function $y=\frac{x-1}{x^2-3x+3}$ transforms the real line is

a) $(0,\infty)$ b) $(-\infty,\infty)$ c) $[0,1]$ d) $[-1/3,1]-\{0\}$

263. The domain of definition of the function $f(x)=x^{\frac{1}{\log_{10}x}}$, is

a) $(0,1)\cup(1,\infty)$ b) $(0,\infty)$ c) $[0,\infty)$ d) $[0,1)\cup(1,\infty)$

264. Let W denotes the words in the English dictionary. Define the relation R by $R=\{(x,y)\in W\times W:$ the world x and y have at least one letter in common$\}$. Then, R is

a) Reflexive, symmetric and not transitive
b) Reflexive, symmetric and transitive
c) Reflexive, not symmetric and transitive
d) Not reflexive, symmetric and transitive

265. The function $f:C\to C$ defined by $f(x)=\frac{ax+b}{cx+d}$ for $x\in C$ where $bd\neq 0$ reduces to a constant function, if

a) $a=c$ b) $b=d$ c) $ad=bc$ d) $ab=cd$

266. Let $A=\{x,y,z\}, B=\{u,v,\omega\}$ and $f:A\to B$ be defined by $f(x)=u, f(y)=v, f(z)=\omega$. Then, f is

a) Surjective but not injective
b) Injective but not surjective
c) Bijective
d) None of these

267. Consider the following relations $R=\{(x,y)\mid$ x, y are real numbers and x=wy for some rational number w$\}$; $S=\left\{\left(\frac{m}{n},\frac{p}{q}\right)\mid m,n,p \text{ and } q \text{ are integers such that } n,q\neq 0 \text{ and } qm=pn\right\}$. Then

a) R is an equivalence relation but S is not an equivalence relation
b) Neither R nor S is an equivalence relation
c) S is an equivalence relation but R is not an equivalence relation
d) R and S both are equivalence relations

268. Which of the following functions has period π ?

a) $|-\tan x|+\cos 2x$
b) $2\sin\frac{\pi x}{3}+3\cos\frac{2\pi x}{3}$
c) $6\cos\left(2\pi x+\frac{\pi}{4}\right)+5\sin\left(\pi x+\frac{3\pi}{4}\right)$
d) $|\tan 2x|+|\sin 4x|$

269. The range of the function $f(x)=\sqrt{(x-1)(3-x)}$ is

a) [0, 1] b) (-1, 1) c) (-3, 3) d) (-3, 1)

270. Let $A=\{x,y,z\}$ and $B=\{a,b,c,d\}$. Which one of the following is not a relation from A to B?

a) $\{(x,a),(x,c)\}$ b) $\{(y,c),(y,d)\}$ c) $\{(z,a),(z,d)\}$ d) $\{(z,b),(y,b),(a,d)\}$

271. If $f(x)$ defined on $[0,1]$ by the rule

$$f(x)=\begin{cases} x, \text{ if } x \text{ is rational} \\ 1-x, \text{ if } x \text{ is irrational}\end{cases}$$

Then, for all $x\in[0,1]$, $f(f(x))$ is

a) Constant b) $1+x$ c) x d) None of these

272. Let $f(x)=\min\{x,x^2\}$, for every $x\in R$. Then,

a) $f(x)=\begin{cases} x, & x\geq 1 \\ x^2, & 0\leq x<1 \\ x, & x<0\end{cases}$

b) $f(x)=\begin{cases} x^2, & x\geq 1 \\ x, & x<1\end{cases}$

c) $f(x)=\begin{cases} x, & x\geq 1 \\ x^2, & x<1\end{cases}$

d) $f(x)=\begin{cases} x^2, & x\geq 1 \\ x, & 0\leq x<1 \\ x^2, & x<0 \end{cases}$

273. If X = {1,2,3,4}, then one-one onto mappings $f: X\to X$ such that $f(1)=1, f(2)\neq 2, f(4)\neq 4$ are given by

a) $f=\{(1,1),(2,3),(3,4),(4,2)\}$

b) $f=\{(1,2),(2,4),(3,3),(4,2)\}$

c) $f=\{(1,2),(2,4),(3,2),(4,3)\}$

d) None of these

274. The domain of the function $f(x)=\exp(\sqrt{5x-3-2x^2})$ is

a) $[3/2,\infty)$ b) $[1,3/2]$ c) $(-\infty,1)$ d) $(1,3/2)$

275. $f(x)=x+\sqrt{x^2}$ is a function from R to$R$, then $f(x)$ is

a) Injective b) Surjective c) Bijective d) None of these

276. If $f(x)=\frac{\sin^4 x+\cos^2 x}{\sin^2 x+\cos^4 x}$ for $x\in R$, then $f(2010)=$

a) 1 b) 2 c) 3 d) 4

277. If $b^2-4ac=0, a>0$, then the domain of the function $f(x)=\log\{ax^3+(a+b)x^2+(b+c)x+c\}$ is

a) $R-\left\{-\frac{b}{2a}\right\}$

b) $R-\left\{\left\{-\frac{b}{2a}\right\}\cup\{x|x\geq -1\}\right\}$

c) $R-\left\{\left\{-\frac{b}{2a}\right\}\cap(-\infty,-1]\right\}$

d) None of these

278. The inverse of the function $y=\frac{10^x-10^{-x}}{10^x+10^{-x}}$ is

a) $\frac{1}{2}\log_{10}\left(\frac{1+x}{1-x}\right)$ b) $\frac{1}{2}\log_{10}\left(\frac{2+x}{2-x}\right)$ c) $\frac{1}{2}\log_{10}\left(\frac{1-x}{1+x}\right)$ d) None of these

279. If $f: R\to R$ is given by

$f(x)=\begin{cases}-1, \text{when } x \text{ is rational} \\ 1, \text{when } x \text{ is irrational}\end{cases}$'

Then $(fof)(1-\sqrt{3})$ is equal to

a) 1 b) -1 c) $\sqrt{3}$ d) 0

280. The function $f: R\to R$ defined by $f(x)=6^x+6^{|x|}$, is

a) One-one and onto b) Many one and onto c) One-one and into d) Many one and into

281. Let $f: N\to Y$ be a function defined as $f(x)=4x+3$ where $Y=\{y\in N: y=4x+3 \text{ for some } x\in N\}$. Show that f is invertible and its inverse is

a) $g(y)=\frac{y-3}{4}$ b) $g(y)=\frac{3y+4}{3}$ c) $g(y)=4+\frac{y+3}{4}$ d) $g(y)=\frac{y+3}{4}$

282. If $f(x)=\sqrt{\cos(\sin x)}+\sqrt{\sin(\cos x)}$, then range of $f(x)$ is

a) $[\sqrt{\cos 1},\sqrt{\sin 1}]$ b) $[\sqrt{\cos 1}, 1+\sqrt{\sin 1}]$ c) $[1-\sqrt{\cos 1},\sqrt{\sin 1}]$ d) None of these

283. Let $f: A\to B$ and g: $B\to C$ be two functions such that g$of: A\to C$ is onto and g is one-one. Then,

a) f is one-one

b) f is onto

c) f is both one-one and onto

d) None of these

284. Let $f:(e,\infty)\to R$ be defined by $f(x)=\log[\log(\log x)]$, then

a) f is one-one but not onto

b) f is onto but not one-one

c) f is both one-one and onto

d) f is neither one-one nor onto

285. If $f: [-6, 6] \to R$ is defined by $f(x) = x^2 - 3$ for $x \in R$, then $(fofof)(-1) + (fofof)(0) + (fofof)(1)$ is equal to

a) $f(4\sqrt{2})$ b) $f(3\sqrt{2})$ c) $f(2\sqrt{2})$ d) $f(\sqrt{2})$

286. Let $f: R = \{n\} \to R$ be a function defined by $f(x) = \frac{x-m}{x-n}$, where $m \neq n$. Then,

a) f is one-one onto b) f is one-one into c) f is many one onto d) f is may one into

287. Let $f(x) = x, g(x) = 1/x$ and $h(x) = f(x)g(x)$. Then, $h(x) = 1$, if

a) x is any rational number
b) x is a non-zero real number
c) x is a real number
d) x is a rational number

288. Which of the following is not periodic?

a) $|\sin 3x| + \sin^2 x$ b) $\cos\sqrt{x} + \cos^2 x$ c) $\cos 4x + \tan^2 x$ d) $\cos 2x + \sin x$

289. If $f(x) = 2^x$, then $f(0), f(1), f(2), \ldots$ are in

a) AP b) GP c) HP d) Arbitrary

290. If $f(\sin x) - f(-\sin x) = x^2 - 1$ is defined for all $x \in R$, then the value of $x^2 - 2$ can be

a) 0 b) 1 c) 2 d) -1

291. If $x \in R$, then $f(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is equal to

a) $2\tan^{-1} x$
b) $\begin{cases} 2\tan^{-1} x, x \geq 0 \\ -2\tan^{-1} x, \ x \leq 0 \end{cases}$
c) $\begin{cases} \pi + 2\tan^{-1} x, \ x \geq 0 \\ -\pi + 2\tan^{-1} x, \ x \leq 0 \end{cases}$
d) None of these

292. Domain of the function $f(x) = \sin^{-1}(\log_2 x)$ in the set of real numbers is

a) $\{x: 1 \leq x \leq 2\}$ b) $\{x: 1 \leq x \leq 3\}$ c) $\{x: -1 \leq x \leq 2\}$ d) $\left\{x: \frac{1}{2} \leq x \leq 2\right\}$

293. If $f: R \to R$ and $g: R \to R$ are given by $f(x) = |x|$ and $g(x) = [x]$ for each $x \in R$, then $\{x \in R : g(f(x)) \leq f(g(x))\} =$

a) $Z \cup (-\infty, 0)$ b) $(-\infty, 0)$ c) Z d) R

294. If $f(x) = \log\left(\frac{1+x}{1-x}\right), -1 < x < 1$, then $f\left(\frac{3x+x^3}{1+3x^2}\right) - f\left(\frac{2x}{1+x^2}\right)$ is

a) $[f(x)]^3$ b) $[f(x)]^2$ c) $-f(x)$ d) $f(x)$

295. The domain of definition of $f(x) = \underbrace{\log_{10}\log_{10}\log_{10}\ldots\log_{10}}_{\to n \text{ times} \leftarrow} x$, is

a) $(10^n, \infty)$ b) $(10^{n-1}, \infty)$ c) $(10^{n-2}, \infty)$ d) None of these

296. The domain of $\sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$ is

a) [1, 9] b) [-1, 9] c) [-9, 1] d) [-9, -1]

297. Domain of definition of the function $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$, is

a) $(1, 2)$ b) $(-1, 0) \cup (1, 2)$
c) $(1, 2) \cup (2, \infty)$ d) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

298. If X and Y are two non-empty sets where $f: X \to Y$ is function is defined such that
$f(C) = \{f(x): x \in C\}$ for $C \subseteq X$
And $f^{-1}(D) = \{x: f(x) \in D\}$ for $D \subseteq Y$,
For any $A \subseteq X$ and $B \subseteq Y$, then

a) $f^{-1}(f(A)) = A$ b) $f^{-1}(f(A)) = A$ only if $f(X) = Y$
c) $f(f^{-1}(B)) = B$ only if $B \subseteq f(x)$ d) $f(f^{-1}(B)) = B$

299. If $f(-x) = -f(x)$, then $f(x)$ is

a) An even function b) An odd function c) Neither odd nor even d) Periodic function

300. If $f: [-2, 2] \to R$ is defined by

$f(x) = \begin{cases} -1, \text{for } -2 \le x \le 0 \\ x - 1, \text{for } 0 \le x \le 2 \end{cases}$

Then $\{x \in [-2,2]: x \le 0 \text{ and } f(|x|) = x\} =$

a) $\{-1\}$ b) $\{0\}$ c) $\{-1/2\}$ d) ϕ

301. If $2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1$ for all $x \in R - \{0\}$, then $f(x^4)$ is

a) $\frac{(1 - x^4)(2x^4 + 3)}{5x^4}$ b) $\frac{(1 + x^4)(2x^4 - 3)}{5x^4}$ c) $\frac{(1 - x^4)(2x^4 - 3)}{5x^4}$ d) None of these

302. The domain of definition of the function $f(x) = {}^{7-x}P_{x-3}$, is

a) $[3, 7]$ b) $\{3, 4, 5, 6, 7\}$ c) $\{3, 4, 5\}$ d) None of these

303. Let $f(x) = x$ and $g(x) = |x|$ for all $x \in R$. Then, the function $\phi(x)$ satisfying $\{\phi(x) - f(x)\}^2 + \{\phi(x) - g(x)\}^2 = 0$, is

a) $\phi(x) = x, x \in [0, \infty)$

b) $\phi(x) = x, x \in R$

c) $\phi(x) = -x, x \in (-\infty, 0]$

d) $\phi(x) = x + |x|, x \in R$

304. The value of the function $f(x) = 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$ lies in the interval

a) $[-\pi/4, \pi/4]$ b) $[0, 3/\sqrt{2}]$ c) $(-3, 3)$ d) None of these

305. The period of the function $f(x) = |\sin x| + |\cos x|$ is

a) π b) $\pi/2$ c) 2π d) None of these

306. If $f(x) = (ax^2 + b)^3$, then the function g such that $f(g(x)) = g(f(x))$ is given by

a) $g(x) = \left(\frac{b - x^{1/3}}{a}\right)^{1/2}$ b) $g(x) = \frac{1}{(a\,x^2 + b)^3}$ c) $g(x) = (a\,x^2 + b)^{1/3}$ d) $g(x) = \left(\frac{x^{1/3} - b}{a}\right)^{1/2}$

307. Let R be the real line. Consider the following subsets of the plane $R \times R$

$S = \{(x, y): y = x + 1 \text{ and } o < x < 2\}$

$T = \{(x, y): x - y \text{ is an integer}\}$

Which of the following is true?

a) T is an equivalent relation on R but S is not b) Neither S nor T is an equivalence relation on R

c) Both S and T are equivalence relations on R d) S is an equivalence relations on R and T is not

308. Let $A = [-1, 1]$ and $f: A \to A$ be defined as $f(x) = x|x|$ for all $x \in A$, then $f(x)$ is

a) Many-one into function b) One-one into function

c) Many-one onto function d) One-one onto function

309. If $f(x) = \frac{1-x}{1+x}, x \ne 0, -1$ and $\alpha = f(f(x)) + f\left(f\left(\frac{1}{x}\right)\right)$, then

a) $\alpha > 2$ b) $\alpha < -2$ c) $|\alpha| > 2$ d) $\alpha = 2$

310. Let R and S be two non-void relations on a set$A$. Which of the following statements is false?

a) R and S are transitive implies $R \cap S$ is transitive.

b) R and S are transitive implies $R \cup S$ is transitive.

c) R and S are symmetric implies $R \cup S$ is symmetric.

d) R and S are reflexive implies $R \cap S$ is reflexive.

311. $A = \{1, 2, 3, 4\}, B\{1, 2, 3, 4, 5, 6\}$are two sets, and function $f: A \to B$ is defined by $f(x) = x + 2\ \forall x \in A$, then the function f is

a) Bijective b) Onto c) One-one d) Many-one

312. Let $f(x) = x + 1$ and $\phi(x) = x - 2$. Then the values of x satisfying $|f(x) + \phi(x)| = |f(x)| + |\phi(x)|$ are :

a) $(-\infty, 1]$ b) $[2, \infty)$ c) $(-\infty, -2]$ d) $[1, \infty)$

313. The domain of the function $f(x) = \frac{\sin^{-1}(3-x)}{\log_e(|x|-2)}$, is

a) $[2,4]$ b) $(2,3)\cup(3,4]$ c) $[2,3)$ d) $(-\infty,-3)\cup[2,\infty)$

314. If $f(x)=\frac{1}{\sqrt{|x|-x}}$ then, domain of $f(x)$ is

a) $(-\infty,0)$ b) $(-\infty,2)$ c) $(-\infty,\infty)$ d) None of the above

315. The domain of definition of
$f(x)=\log_{10}\{(\log_{10}x)^2-5\log_{10}x+6\}$, is

a) $(0,10^2)$ b) $(10^3,\infty)$ c) $(10^2,10^3)$ d) $(0,10^2)\cup(10^3,\infty)$

316. If a function $f(x)$ satisfies the condition
$f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2}, x\neq 0$, then $f(x)$ equals

a) x^2-2 for all $x\neq 0$
b) x^2-2 for all x satisfying $|x|\geq 2$
c) x^2-2 for all x satisfying $|x|<2$
d) None of these

317. The period of the function $f(x)=\sin\left(\frac{2x+3}{6\pi}\right)$, is

a) 2π b) 6π c) $6\pi^2$ d) None of these

318. $f:R\to R$ is a function defined by $f(x)=10x-7$. If $g=f^{-1}$, then $g(x)=$

a) $\frac{1}{10x-7}$ b) $\frac{1}{10x+7}$ c) $\frac{x+7}{10}$ d) $\frac{x-7}{10}$

319. If $f(x)=[x-2]$, where $[x]$ denotes the greatest integer less than or equal to x, then $f(2,5)$ is equal to

a) $\frac{1}{2}$ b) 0 c) 1 d) Does not exist

320. The domain of definition of
$f(x)=\sqrt{\log_{10}(\log_{10}x)-\log_{10}(4-\log_{10}x)-\log_{10}3}$, is

a) $(10^3,10^4)$ b) $[10^3,10^4]$ c) $[10^3,10^4)$ d) $(10^3,10^4]$

321. The value of $n\in Z$ (the set of integers) for which the function $f(x)=\sin\frac{\sin nx}{\sin\left(\frac{x}{n}\right)}$ has 4π as its period is

a) 2 b) 3 c) 5 d) 4

322. The inverse of the function $f:R\to R$ given by $f(x)=\log_a\left(x+\sqrt{x^2+1}\right)$ $(a>0, a\neq 1)$, is

a) $\frac{1}{2}(a^x+a^{-x})$ b) $\frac{1}{2}(a^x-a^{-x})$ c) $\frac{1}{2}\left(\frac{a^x+a^{-x}}{a^x-a^{-x}}\right)$ d) Not defined

323. The domain of definition of the function
$f(x)=x\cdot\frac{1+2(x+4)^{-0.5}}{2-(x+4)^{0.5}}+(x+4)^{0.5}+4(x+4)^{0.5}$ is

a) R b) $(-4,4)$ c) R^+ d) $(-4,0)\cup(0,\infty)$

324. If $f(x)=\frac{\alpha x}{x+1}, x\neq -1$, for what value of α is $f[f(x)]=x$?

a) $\sqrt{2}$ b) $-\sqrt{2}$ c) 1 d) -1

325. The period of the function $f(x)=\operatorname{cosec}^2 3x+\cot 4x$ is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{6}$ d) π

326. The domain of the definition of the function $f(x)=\sqrt{1+\log_e(1-x)}$ is

a) $-\infty<x\leq 0$ b) $-\infty<x\leq\frac{e-1}{e}$ c) $-\infty<x\leq 1$ d) $x\geq 1-e$

327. The range of the function $\sin(\sin^{-1}x+\cos^{-1}x), |x|\leq 1$ is

a) [-1, 1] b) [1, -1] c) {0} d) {1}

328. The range of $f(x)=\cos x-\sin x$ is

a) [-1, 1] b) (-1, 2) c) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ d) $[-\sqrt{2},\sqrt{2}]$

329. The range of function $f(x)=x^2+\frac{1}{x^2+1}$

a) $[1, \infty)$ b) $[2, \infty)$ c) $\left[\frac{3}{2}, \infty\right)$ d) None of these

330. If n is an integer, the domain of the function $\sqrt{\sin 2x}$ is

a) $\left[n\pi - \frac{\pi}{2}, n\pi\right]$ b) $\left[n\pi, n\pi + \frac{\pi}{4}\right]$ c) $[(2n-1)\pi, 2n\pi]$ d) $[2n\pi, (2n+1)\pi]$

331. If $f : R \rightarrow R$ is defined by $f(x) = x - [x] - \frac{1}{2}$ for all $x \in R$, where $[x]$ denotes the greatest integer function, then $\left\{x \in R : f(x) = \frac{1}{2}\right\}$ is equal to

a) Z b) N c) ϕ d) R

332. Suppose $f: [-2, 2] \rightarrow R$ is defined by

$f(x) = \begin{cases} -1, \text{for } -2 \le x \le 0 \\ x - 1 \text{ for } 0 \le x \le 2 \end{cases}$, then $\{x \in [-2, 2]: x \le 0 \text{ and } f(|x|) = x\}$ is equal to

a) $\{-1\}$ b) $\{0\}$ c) $\left\{-\frac{1}{2}\right\}$ d) ϕ

333. If $f: R \rightarrow R$ is defined by $f(x) = \sin x$ and $g: (1, \infty) \rightarrow R$ is defined by $g(x) = \sqrt{x^2 - 1}$, then $gof(x)$ is

a) $\sqrt{\sin(x^2 - 1)}$ b) $\sin\sqrt{x^2 - 1}$ c) $\cos x$ d) Not defined

334. Let R and C denote the set of real numbers and complex numbers respectively. The function $f: C \rightarrow R$ defined by $f(z) = |z|$ is

a) One to one
b) Onto
c) Bijective
d) Neither one to one nor onto

335. If $f(x) = \frac{x-1}{x+1}$, then $f(2x)$ is

a) $\frac{f(x)+1}{f(x)+3}$ b) $\frac{3f(x)+1}{f(x)+3}$ c) $\frac{f(x)+3}{f(x)+1}$ d) $\frac{f(x)+3}{3f(x)+1}$

336. The range of the function $f(x) = \tan\sqrt{\frac{\pi^2}{9} - x^2}$ is

a) $[0, 3]$ b) $[0, \sqrt{3}]$ c) $(-\infty, \infty)$ d) None of these

337. The domain of the function $f(x) = \operatorname{cosec}^{-1}[\sin x]$ in $[0, 2\pi]$, where $[\cdot]$ denotes the greatest integer function, is

a) $[0, \pi/2) \cup (\pi, 3\pi/2]$ b) $(\pi, 2\pi) \cup \{\pi/2\}$ c) $(0, \pi] \cup \{3\pi/2\}$ d) $(\pi/2, \pi) \cup (3\pi/2, 2\pi)$

338. Let R be the relation on the set R of all real numbers defined by aRb if $|a - b| \le 1$, then R is

a) Reflexive and symmetric
b) Symmetric only
c) Transitive only
d) Anti-symmetric only

339. The domain of the function $f(x) = \log_e(x - [x])$ is

a) R b) $R - Z$ c) $(0, +\infty)$ d) Z

340. If $f: [0, \infty] \rightarrow [0, \infty]$ and $f(x) = \frac{x}{1+x}$, then f is

a) One-one and onto
b) One-one but not onto
c) Onto but not one-one
d) Neither one-one nor onto

341. The function $f: R \rightarrow R$ given by $f(x) = x^3 - 1$ is

a) A one-one function
b) An onto function
c) A bijection
d) Neither one-one nor onto

342. Let $[x]$ denote the greatest integer $\le x$. If $f(x) = [x]$ and $g(x) = |x|$, then the value of $f\left(g\left(\frac{8}{5}\right)\right) - g\left(f\left(-\frac{8}{5}\right)\right)$ is

a) 2 b) -2 c) 1 d) -1

343. The domain of the function $f(x) = \frac{\cos^{-1} x}{[x]}$ is

a) $[-1, 0) \cup \{1\}$ b) [-1, 1] c) [-1, 1) d) None of these

344. The set of values of x for which of the function $f(x) = \frac{1}{x} + 2^{\sin^{-1} x} + \frac{1}{\sqrt{x-2}}$ exists is

a) R b) $R - \{0\}$ c) ϕ d) None of these

345. If $f(x)$ satisfies the relation $2f(x) + f(1-x) = x^2$ for all real x, then $f(x)$ is

a) $\frac{x^2+2x-1}{6}$ b) $\frac{x^2+2x-1}{3}$ c) $\frac{x^2+4x-1}{3}$ d) $\frac{x^2-3x+1}{6}$

346. If the function $f(x)$ is defined by $f(x) = a + bx$ and $f^r = fff$.... (repeated r times), then $f^r(x)$ is equal to

a) $a + b^r x$ b) $ar + b^r x$ c) $ar + bx^r$ d) $a\left(\frac{b^r-1}{b-1}\right) + b^r x$

347. If $f(x) = \frac{x-1}{x+1}$, then $f(2x)$ is

a) $\frac{f(x)+1}{f(x)+3}$ b) $\frac{3f(x)+1}{f(x)+3}$ c) $\frac{f(x)+3}{f(x)+1}$ d) $\frac{f(x)+3}{3f(x)+1}$

348. If $f(x)$ is an odd periodic function with period 2, then $f(4)$ equals

a) 0 b) 2 c) 4 d) -4

349. The domain of definition of

$f(x) = \sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right) \times \frac{1}{x^2-36}}$, is

a) $(-\infty, 0) - \{-6\}$ b) $(0, \infty) - \{1, 6\}$ c) $(1, \infty) - \{6\}$ d) $[1, \infty) - \{6\}$

350. The domain of the function $f(x) = \log_2(\log_3(\log_4 x))$is

a) $(-\infty, 4)$ b) $(4, \infty)$ c) $(0, 4)$ d) $(1, \infty)$

351. Let $f(x) = |x-2| + |x-3| + |x-4|$ and $g(x) = x + 1$. Then,

a) $g(x)$ is an even function

b) $g(x)$ is an odd function

c) $g(x)$ is neither even nor odd

d) $g(x)$ is periodic

352. If a function $f : [2, \infty) \to B$ defined by $f(x) = x^2 - 4x + 5$ is a bijection, then $B =$

a) R b) $[1, \infty)$ c) $[4, \infty)$ d) $[5, \infty)$

353. R is relation on N given by $R = \{(x, y): 4x + 3y = 20\}$. Which of the following belongs to R?

a) (-4, 12) b) (5, 0) c) (3, 4) d) (2, 4)

354. If $f: R \to R$ be a mapping defined by $f(x) = x^3 + 5$, then $f^{-1}(x)$ is equal to

a) $(x+5)^{1/3}$ b) $(x-5)^{1/3}$ c) $(5-x)^{1/3}$ d) $5 - x$

355. Let $f(x) = x$ and $g(x) = |x|$ for all $x \in R$. Then, the function $\phi(x)$ satisfying $[\phi(x) - f(x)]^2 + [\phi(x) - g(x)]^2 = 0$

a) $\phi(x) = x, x \in [0, \infty)$

b) $\phi(x) = x, x \in R$

c) $\phi(x) = -x, x \in (-\infty, 0]$

d) $\phi(x) = x + |x|, x \in R$

356. In a function $f(x)$ is defined for $x \in [0, 1]$, then the function $f(2x+3)$ is defined for

a) $x \in [0, 1]$ b) $x \in [-3/2, -1]$ c) $x \in R$ d) $x \in [-3/2, 1]$

357. If $f(x) = x^2 - 2|x|$ and

$g(x) = \begin{cases} \text{Min}\{f(t) : -2 \le t \le x\}, -2 \le x < 0 \\ \text{Max}\{f(t) : 0 \le\le x\}, \ 0 \le x \le 3 \end{cases}$, then $g(x)$ equlas

a) $\begin{cases} x^2 - 2x, & -2 \le x \le -1 \\ -1, & -1 \le x < 0 \\ 0, & 0 \le x < 2 \\ x^2 + 2x, & 2 \le x \le 3 \end{cases}$

b) $\begin{cases} x^2 + 2x, & -2 \le x \le -1 \\ -1, & -1 \le x < 0 \\ 0, & 0 \le x < 1 \\ x^2 - 2x, & 1 \le x \le 3 \end{cases}$

c) $\begin{cases} x^2 + 2x, & -2 \le x \le -0 \\ x^2 - 2x, & 0 \le x \le 3 \end{cases}$

d) $\begin{cases} x^2+2x, & -2 \le x \le 0 \\ 0, & 0 \le x < 2 \\ x^2-2x, & 2 \le x \le 3 \end{cases}$

358. Let R be the set of real numbers and the mapping $f: R \to R$ and $g: R \to R$ be defined by $f(x) = 5 - x^2$ and $g(x) = 3x - 4$, then the value of $(fog)(-1)$ is

a) -44 b) -54 c) -32 d) -64

359. $f: R \to R$ is defined by $f(x) = \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$, is

a) One-one but not onto

b) Many-one but onto

c) One-one and onto

d) Neither one-one nor onto

360. Let $f: N \to N$ defined by $f(x) = x^2 + x + 1, x \in N$, then f is

a) One-one onto b) Many-one onto c) One -one but not onto d) None of these

361. Which of the following functions have period 2π?

a) $y = \sin\left(2\pi t + \frac{\pi}{3}\right) + 2\sin\left(3\pi t + \frac{\pi}{4}\right) + 3\sin 5\pi t$ b) $y = \sin\frac{\pi}{3}t + \sin\frac{\pi}{4}t$

c) $y = \sin t + \cos 2t$ d) None of the above

362. Let $f: A \to B$ be a function defined by $f(x) = \sqrt{3}\sin x + \cos x + 4$. If f is invertible, then

a) $A = [-2\pi/3, \pi/3], B = [2, 6]$

b) $A = [\pi/6, 5\pi/6], B = [-2, 2]$

c) $A = [-\pi/2, \pi/2], B = [2, 6]$

d) $A = [-\pi/3, \pi/3], B = [2, 6]$

363. If $f: R \to R$ and $g: R \to R$ are defined by $f(x) = 2x + 3$ and $g(x) = x^2 + 7$, then the values of x such that $g(f(x)) = 8$ are

a) $1, 2$ b) $-1, 2$ c) $-1, -2$ d) $1, -2$

364. The domain of definition of the function

$f(x) = \sin^{-1}\left(\frac{x-3}{2}\right) - \log_{10}(4 - x)$, is

a) $1 \le x \le 5$ b) $1 < x < 4$ c) $1 \le x < 4$ d) $1 \le x \le 4$

365. If $f(x) = \frac{1-x}{1+x}(x \neq -1)$, then $f^{-1}(x)$ equals to

a) $f(x)$ b) $\frac{1}{f(x)}$ c) $-f(x)$ d) $-\frac{1}{f(x)}$

366. The function f satisfies the functional equation $3f(x) + 2f\left(\frac{x+59}{x-1}\right) = 10x + 30$ for all real $x \neq 1$. The value of $f(7)$ is

a) 8 b) 4 c) -8 d) 11

367. If $[x]$ denotes the greatest integer $\le x$, then

$\left[\frac{2}{3}\right] + \left[\frac{2}{3} + \frac{1}{99}\right] + \left[\frac{2}{3} + \frac{2}{99}\right] + \ldots + \left[\frac{2}{3} + \frac{98}{99}\right]$ is equal to

a) 99 b) 98 c) 66 d) 65

368. If $f(x)$ is defined on $[0, 1]$, then the domain of $f(3x^2)$, is

a) $[0, 1/\sqrt{3}]$ b) $[-1/\sqrt{3}, 1/\sqrt{3}]$ c) $[-\sqrt{3}, \sqrt{3}]$ d) None of these

369. If $f: R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x - 1$, is onto, then the intervel of s is

a) [0, 3] b) [-1, 1] c) [0, 1] d) [-1, 3]

370. If $f(x) = e^x$ and $g(x) = \log_e x$, then which of the following is true?

a) $f\{g(x)\} \neq g\{f(x)\}$ b) $f\{g(x)\} = g\{f(x)\}$

c) $f\{g(x)\} + g\{f(x)\} = 0$ d) $f\{g(x)\} - g\{f(x)\} = 1$

371. The range of the function $f(x) = {}^{7-x}P_{x-3}$, is

a) $\{1, 2, 3\}$ b) $\{1, 2, 3, 4, 5, 6\}$ c) $\{1, 2, 3, 4\}$ d) $\{1, 2, 3, 4, 5\}$

372. The domain of definition of $f(x)=\log_{1.7}\left(\frac{2-\phi'(x)}{x+1}\right)^{1/2}$, where $\phi(x)=\frac{x^3}{3}-\frac{3}{2}x^2-2x+\frac{3}{2}$, is

a) $(-\infty,-4)$ b) $(-4,\infty)$ c) $(-\infty,-1)\cup(-1,4)$ d) $(-\infty,-1)\cup(-1,4]$

373. The domain of definition of the function

$f(x)=\sin^{-1}\left(\frac{4}{3+2\cos x}\right)$, is

a) $\left[2n\pi-\frac{\pi}{6},2n\pi+\frac{\pi}{6}\right], n\in Z$

b) $\left[0,2n\pi+\frac{\pi}{6}\right], n\in Z$

c) $\left[2n\pi-\frac{\pi}{6},0\right], n\in Z$

d) $\left(2n\pi-\frac{\pi}{6},2n\pi+\frac{\pi}{6}\right), n\in Z$

374. Which of the following functions has period 2π?

a) $f(x)=\sin\left(2\pi x+\frac{\pi}{3}\right)+2\sin\left(3\pi x+\frac{\pi}{4}\right)+3\sin 5\pi x$

b) $f(x)=\sin\frac{\pi x}{3}+\sin\frac{\pi x}{4}$

c) $f(x)=\sin x+\cos 2x$

d) None of these

375. Let S be the set of all real numbers. Then, the relation $R=\{(a,b):1+ab>0\}$ on S is

a) Reflexive and symmetric but not transitive b) Reflexive and transitive but not symmetric

c) Symmetric and transitive but not reflexive d) Reflexive, transitive and symmetric

376. Which of the following functions is periodic?

a) $f(x)=x+\sin x$ b) $f(x)=\cos\sqrt{x}$ c) $f(x)=\cos x^2$ d) $f(x)=\cos^2 x$

377. The function $f(x)=\max\{(1-x),(1+x),2\}, x\in(-\infty,\infty)$ is equivalent to

a) $f(x)=\begin{cases}1-x, & x\le -1\\ 2, & -1<x<1\\ 1+x, & x\ge 1\end{cases}$

b) $f(x)=\begin{cases}1+x, & x\le -1\\ 2, & -1<x<1\\ 1-x, & x\ge 1\end{cases}$

c) $f(x)=\begin{cases}1-x, & x\le -1\\ 1, & -1<x<1\\ 1+x, & x\ge 1\end{cases}$

d) None of these

378. The period of the function $f(\theta)=\sin\frac{\theta}{3}+\cos\frac{\theta}{2}$ is

a) 3π b) 6π c) 9π d) 12π

379. Let the function $f(x)=x^2+x+\sin x-\cos x+\log(1+|x|)$ be defined on the interval $[0,1]$. The odd extension of $f(x)$ to the interval $[-1,1]$ is

a) $x^2+x+\sin x+\cos x-\log(1+|x|)$

b) $-x^2+x+\sin x+\cos x-\log(1+|x|)$

c) $-x^2+x+\sin x-\cos x+\log(1+|x|)$

d) None of these

380. If $g(x)=1+\sqrt{x}$ and $f(g(x))=3+2\sqrt{x}+x$ then, $f(x)$ is equal to

a) $1+2x^2$ b) $2+x^2$ c) $1+x$ d) $2+x$

381. Let $f:(-1,1)\to B$, be a function defined by $f(x)=\tan^{-1}\frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval

a) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ b) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ c) $\left[0,\frac{\pi}{2}\right)$ d) $\left(0,\frac{\pi}{2}\right)$

382. If $f:R\to R$ defined by $f(x)=x^3$, then $f^{-1}(8)$ is equal to

a) $\{2\}$ b) $\{2,\omega,2\omega^2\}$ c) $\{2,-2\}$ d) $\{2,2\}$

383. The set of all x for which there are no functions

$f(x) = \log_{(x-2)/(x+3)} 2$ and $g(x) = \frac{1}{\sqrt{x^2-9}}$, is

a) $[-3, 2]$ b) $[-3, 2)$ c) $(-3, 2]$ d) $(-3, -2)$

384. Which of the following functions is (are) not an injective map(s)?

a) $f(x) = |x+1|, x \in [-1, \infty)$

b) $g(x) = x + \frac{1}{x}, x \in (0, \infty)$

c) $h(x) = x^2 + 4x - 5, x \in (0, \infty)$

d) $k(x) = e^{-x}, x \in [0, \infty)$

385. If $f: N \to Z$ is defined by

$$f(n) = \begin{cases} 2 & \text{if } n = 3k, k \in Z \\ 10 & \text{if } n = 3k+1, k \in Z, \\ 0 & \text{if } n = 3k+2, k \in Z \end{cases}$$

Then $\{n \in N : f(n) > 2\}$ is equal to

a) $\{3, 6, 4\}$ b) $\{1, 4, 7\}$ c) $\{4, 7\}$ d) $\{7\}$

386. If $f(x) = \frac{2x-1}{x+5} (x \neq -5)$, then $f^{-1}(x)$ is equal to

a) $\frac{x+5}{2x-1}, x \neq \frac{1}{2}$ b) $\frac{5x+1}{2-x}, x \neq 2$ c) $\frac{x-5}{2x+1}, x \neq \frac{1}{2}$ d) $\frac{5x-1}{2-x}, x \neq 2$

387. If a, b are two fixed positive integers such that

$f(a+x) = b + [b^3 + 1 - 3b^2 f(x) + 3b\{f(x)\}^2 - \{f(x)\}^3]^{1/3}$

For all $x \in R$, then $f(x)$ is a periodic function with period

a) a b) $2a$ c) b d) $2b$

388. Let A be a set containing 10 distinct elements, then the total number of distinct function from A to A is

a) 10^{10} b) 101 c) 2^{10} d) $2^{10} - 1$

389. If Q denotes the set of all rational numbers and $f\left(\frac{p}{q}\right) = \sqrt{p^2 - q^2}$ for any $\frac{p}{q} \in Q$, then observe the following statements.

I. $f\left(\frac{p}{q}\right)$ is real for each $\frac{p}{q} \in Q$.

II. $f\left(\frac{p}{q}\right)$ is a complex number for each $\frac{p}{q} \in Q$.

Which of the following is correct?

a) Both I and II are true b) I is true, II is false

c) I is false, II is true d) Both I and II are false

390. The domain of the function $f(x) = \log_{3+x}(x^2 - 1)$ is

a) $(-3, -1) \cup (1, \infty)$ b) $[-3, -1] \cup [1, \infty]$

c) $(-3, -2) \cup (-2, -1) \cup (1, \infty)$ d) $[-3, -2) \cup (-2, -1) \cup (1, \infty)$

391. Let $A = R - \{3\}, B = R - \{1\}$. Let $f: A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$. Then,

a) f is bijective b) f is one-one but not onto

c) f is onto but not one-one d) None of the above

392. Let $f(x) = \frac{\sqrt{\sin x}}{1 + \sqrt[3]{\sin x}}$. If D is the domain of f, then D contains

a) $(0, \pi)$ b) $(-2\pi, -\pi)$ c) $(3\pi, 4\pi)$ d) $(4\pi, 6\pi)$

393. Let $f: R \to R$ and $g: R \to R$ be given by $f(x) = 3x^2 + 2$ and $g(x) = 3x - 1$ for all $x \in R$. Then,

a) $fog(x) = 27x^2 - 18x + 5$

b) $fog(x) = 27x^2 + 18x - 5$

c) $gof(x) = 9x^2 - 5$

d) $gof(x) = 9x^2 + 15$

394. The domain of definition of the function

$f(x) = \dfrac{1}{\sqrt{|x| - x}}$, is

a) R b) $(0, \infty)$ c) $(-\infty, 0)$ d) None of these

395. Let $f: A \to B$ and g: $B \to A$ be two functions such that $fog = I_B$. Then,

a) f and g both are injections

b) f and g both are surjections

c) f is an injection and g is a surjection

d) f is a surjection and g is an injection

396. If $f(x) = x^2 - 1$ and $g(x) = (x+1)^2$, then $(gof)(x)$ is

a) $(x+1)^4 - 1$ b) $x^4 - 1$ c) x^4 d) $(x+1)^4$

397. If $f: R \to R$ satisfies $f(x+y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is

a) $\dfrac{7n}{2}$ b) $\dfrac{7(n+1)}{2}$ c) $7n(n+1)$ d) $\dfrac{7n(n+1)}{2}$

398. If $f(x) = 2x^4 - 13x^2 + ax + b$ is divisible by $x^2 - 3x + 2$, then (a, b) is equal to

a) $(-9, -2)$ b) $(6, 4)$ c) $(9, 2)$ d) $(2, 9)$

399. Let $f: R \to R$ be a function defined by $f(x) = \frac{x^2-8}{x^2+2}$ Then, f is

a) One-one but not onto

b) One-one and onto

c) Onto but not one-one

d) Neither one-one nor onto

400. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$, is

a) $[1, 2)$ b) $[2, 3)$ c) $[1, 2]$ d) $[2, 3]$

RELATIONS AND FUNCTIONS

: ANSWER KEY :

1)	a	2)	a	3)	b	4)	d	153)	d	154)	b	155)	d	156)	c
5)	d	6)	a	7)	c	8)	a	157)	c	158)	c	159)	c	160)	b
9)	c	10)	a	11)	a	12)	c	161)	b	162)	d	163)	a	164)	a
13)	d	14)	c	15)	b	16)	d	165)	b	166)	c	167)	d	168)	c
17)	c	18)	c	19)	b	20)	c	169)	b	170)	b	171)	d	172)	b
21)	c	22)	a	23)	a	24)	d	173)	d	174)	c	175)	b	176)	b
25)	b	26)	d	27)	b	28)	b	177)	b	178)	c	179)	a	180)	d
29)	b	30)	b	31)	a	32)	c	181)	b	182)	d	183)	b	184)	b
33)	a	34)	b	35)	c	36)	c	185)	d	186)	d	187)	b	188)	c
37)	d	38)	c	39)	d	40)	c	189)	b	190)	b	191)	b	192)	a
41)	d	42)	a	43)	c	44)	d	193)	d	194)	c	195)	b	196)	c
45)	b	46)	c	47)	a	48)	c	197)	a	198)	a	199)	a	200)	c
49)	d	50)	d	51)	d	52)	d	201)	a	202)	b	203)	c	204)	c
53)	d	54)	a	55)	b	56)	c	205)	b	206)	b	207)	b	208)	a
57)	d	58)	c	59)	c	60)	c	209)	d	210)	a	211)	c	212)	a
61)	b	62)	d	63)	a	64)	d	213)	d	214)	b	215)	b	216)	c
65)	b	66)	a	67)	c	68)	a	217)	c	218)	c	219)	c	220)	c
69)	b	70)	a	71)	b	72)	c	221)	c	222)	b	223)	b	224)	c
73)	c	74)	d	75)	a	76)	c	225)	b	226)	c	227)	b	228)	d
77)	a	78)	a	79)	d	80)	b	229)	d	230)	c	231)	b	232)	d
81)	a	82)	a	83)	d	84)	b	233)	b	234)	c	235)	d	236)	b
85)	c	86)	a	87)	a	88)	d	237)	c	238)	d	239)	d	240)	b
89)	a	90)	c	91)	a	92)	b	241)	d	242)	c	243)	c	244)	a
93)	c	94)	a	95)	a	96)	a	245)	d	246)	a	247)	d	248)	a
97)	b	98)	d	99)	d	100)	c	249)	b	250)	a	251)	c	252)	a
101)	b	102)	b	103)	c	104)	c	253)	c	254)	d	255)	b	256)	b
105)	b	106)	d	107)	b	108)	b	257)	c	258)	c	259)	d	260)	d
109)	d	110)	d	111)	b	112)	b	261)	a	262)	d	263)	a	264)	a
113)	a	114)	c	115)	c	116)	c	265)	c	266)	c	267)	c	268)	a
117)	b	118)	b	119)	a	120)	c	269)	a	270)	d	271)	c	272)	a
121)	b	122)	b	123)	c	124)	c	273)	a	274)	b	275)	d	276)	a
125)	d	126)	b	127)	b	128)	b	277)	c	278)	a	279)	b	280)	c
129)	a	130)	c	131)	c	132)	b	281)	a	282)	b	283)	b	284)	c
133)	b	134)	c	135)	a	136)	b	285)	a	286)	b	287)	b	288)	b
137)	b	138)	a	139)	d	140)	c	289)	b	290)	d	291)	b	292)	d
141)	d	142)	d	143)	c	144)	d	293)	a	294)	d	295)	d	296)	a
145)	c	146)	a	147)	a	148)	b	297)	d	298)	c	299)	b	300)	c
149)	b	150)	b	151)	d	152)	a	301)	a	302)	c	303)	a	304)	b

305)	b	306)	d	307)	a	308)	d
309)	c	310)	b	311)	c	312)	b
313)	b	314)	a	315)	d	316)	b
317)	c	318)	c	319)	b	320)	c
321)	a	322)	b	323)	d	324)	d
325)	d	326)	b	327)	d	328)	d
329)	a	330)	b	331)	c	332)	c
333)	d	334)	d	335)	b	336)	b
337)	b	338)	a	339)	b	340)	b
341)	c	342)	d	343)	a	344)	c
345)	b	346)	d	347)	b	348)	a
349)	c	350)	b	351)	c	352)	b
353)	d	354)	b	355)	a	356)	b

357)	b	358)	a	359)	a	360)	c
361)	c	362)	a	363)	c	364)	c
365)	a	366)	b	367)	c	368)	b
369)	d	370)	b	371)	a	372)	c
373)	a	374)	c	375)	a	376)	d
377)	a	378)	d	379)	b	380)	b
381)	a	382)	a	383)	d	384)	b
385)	b	386)	b	387)	b	388)	a
389)	c	390)	c	391)	a	392)	a
393)	a	394)	c	395)	d	396)	c
397)	d	398)	c	399)	d	400)	b

RELATIONS AND FUNCTIONS

: HINTS AND SOLUTIONS :

1 **(a)**

We have,

$f(x) = ||x| - 1|$

$\Rightarrow f(x) = \begin{cases} 1 - |x|, \text{if } |x| < 1 \\ |x| - 1, \text{if } |x| \geq 1 \end{cases}$

$\Rightarrow f(x) = \begin{cases} 1 - |x|, \text{if } -1 < x < 1 \\ |x| - 1, \text{if } x \leq -1 \text{ or }, x \geq 1 \end{cases}$

$\Rightarrow f(x) = \begin{cases} 1 + x, \text{if } -1 < x < 0 \\ 1 - x, \text{if } 0 \leq x \leq 1 \\ -x - 1, \text{if } x \leq -1 \\ x - 1, \text{if } x \geq 1 \end{cases}$

2 **(a)**

We have,

$f(x) = \log_{100x}\left(\frac{2\log_{10} x + 1}{-x}\right)$

$f(x)$ is defined if

$x > 0, 100x \neq 1$ and $\frac{2\log_{10} x + 1}{-x} > 0$

$\Rightarrow x > 0, x \neq 10^{-2}$ and $2\log_{10} x + 1 < 0$

$\Rightarrow x < 0, x \neq 10^{-2}$ and $\log_{10} x < -\frac{1}{2}$

$\Rightarrow x > 0, x \neq 10^{-2}$ and $x < 10^{-1/2}$

$\Rightarrow x \in (0, 10^{-2}) \cup (10^{-2} \cup > 10^{-1/2})$

3 **(b)**

The function $f(x)$ will be defined, if

$-1 \leq (x-3) \leq 1 \Rightarrow 2 \leq x \leq 4$

And $9 - x^2 > 0 \Rightarrow -3 < x < 3$

$\therefore 2 \leq x < 3$

4 **(d)**

The given function is

$f(x) = |x| = \begin{cases} x, x \geq 0 \\ x, x < 0 \end{cases}$

And $f: R \to R$, then it is clear that function is neither one-one nor onto.

5 **(d)**

Given, $f(x) = \frac{1}{\sqrt{-x}}$

$\therefore fof(x) = f(f(x)) = f\left(\frac{1}{\sqrt{-x}}\right)$

$\Rightarrow fof(x) = \frac{1}{\sqrt{-\frac{1}{\sqrt{-x}}}}$

Since, $\sqrt{-\frac{1}{\sqrt{-x}}}$ is an imaginary.

Hence, no domain of $fof(x)$ exist.

Thus, the domain of $fof(x)$ is an empty set.

6 **(a)**

We have

$f(x+1) + f(x-1) = \sqrt{2}\, f(x)$ for all $x \in R$ ...(i)

Replacing x by $x+1$ and $x-1$ respectively, we get

$f(x+2) + f(x) = \sqrt{2}\, f(x+1)$ ...(ii)

And,

$f(x) + f(x-2) = \sqrt{2}\, f(x-1)$ ...(iii)

Adding (ii) and (iii) we get

$f(x+2) + f(x-2) + 2f(x)$

$= \sqrt{2}\{f(x+1) + f(x-1)\}$

$f(x+2) + f(x-2) + 2f(x) = \sqrt{2}\{\sqrt{2}f(x)\}$

[Using (i)]

$f(x+2) + f(x-2) + 2f(x) = 2f(x)$

$\Rightarrow f(x+2) + f(x-2) = 0$ for all $x \in R$

Replacing x by $x+2$, we get

$f(x+4) + f(x) = 0 \Rightarrow f(x+4) = -f(x)$ ...(iv)

Replacing x by $x+4$, we get

$f(x+8) = -f(x+4)$ ...(v)

From (iv) and (v), we get

$f(x+8) = f(x)$ for all $x \in R$

Hence, $f(x)$ is periodic with period 8

7 **(c)**

$D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$f(2, 5, 15) = (2+5).(5' + 15)$

$= 10.\left(\frac{30}{5} + 15\right)$

$\left(\because 2 + 5 = \text{LCM of } (2,5) = 10 \text{ and } 5' = \frac{30}{5}\right)$

$= 10(6 + 15) = 10.30 = 10$

8 **(a)**

For $f(x)$ to be defined

$$\frac{5x-x^2}{4} \geq 1 \Rightarrow x^2-5x+4 \leq 0$$

$\Rightarrow (x-4)(x-1) \leq 0 \therefore x \in [1,4]$

9 **(c)**

In the given options only option (c) satisfies the condition of a function.

Hence, option (c) is a function.

10 **(a)**

We have,

$f(x)=2x-3$ and $g(x)=x^3+5$

Clearly, $f: R \to R$ and $g: R \to R$ are bijections.

Therefore, $fog : R \to R$ is also a bijection and hence invertible

Now,

$$fog\,(x)=f(g(x))=f(x^3+5)=2(x^3+5)-3$$
$$=2x^3+7$$

Let $h(x)=fog\,(x)$. Then, $h(x)=2x^3+7$

Now,

$hoh^{-1}(x)=x$

$\Rightarrow h(h^{-1}(x))=x$

$$\Rightarrow 2\{h^{-1}(x)\}^3+7=x \Rightarrow h^{-1}(x)=\left(\frac{x-7}{2}\right)^{1/3}$$

11 **(a)**

For $x \in (\pi, 3\pi/2)$, we have

$-1 < \sin x < 0$

$\Rightarrow 0 < 1+\sin x < 1$ and $1 < (2+\sin x) < 2$

$\therefore [\sin x]=-1, [1+\sin x]=0$ and $[2+\sin x]=1$

$\Rightarrow f(x)=[\sin x]+[1+\sin x]+[2+\sin x]$
$=-1+0+1=0$

For $x=\pi$, we have

$[\sin x]=0, [1+\sin x]=1$ and $[2+\sin x]=2$

$\therefore f(x)=0+1+2=3$

For $x=\frac{3\pi}{2}$, we have

$[\sin x]=-1, [1+\sin x]=0$ and $[2+\sin x]=1$

$\therefore f(x)=-1+0+1=0$

Hence, range of $f(x)=\{0,3\}$

12 **(c)**

We know that two functions $f(x)$ and $g(x)$ are identical, if their domains are same and $f(x)=g(x)$

Clearly, $f(x)=g(x)$

Now, $D_1=$ Domain $(f)=(3,\infty)$

And, $D_2=$ Domain $(g)=(-\infty,2)\cup(3,\infty)$

$\therefore D_1 \cap D_2=(3,\infty)$

Hence, $f(x)=g(x)$ for all $x \in (3,\infty)$

13 **(d)**

We have,

$1-e^{\frac{1}{x}-1} > 0$

$\Rightarrow e^{\frac{1}{x}-1} < 1 \Rightarrow \frac{1}{x}-1 < 0 \Rightarrow \frac{1}{x} < 1 \Rightarrow x \in (-\infty,0)\cup(1,\infty)$

14 **(c)**

$f(a)=a$

$\Rightarrow \frac{\alpha a^2}{\alpha+1}=a$

$\Rightarrow \alpha a^2=a^2+a$

$\Rightarrow \alpha=1+\frac{1}{a} \qquad (\because a \neq 0)$

15 **(b)**

We have,

$$f(x)=\begin{cases}-1, -2 \leq x \leq 0\\ x-1, 0 < x \leq 2\end{cases}$$

Since $x \in [-2,2]$, therefore $|x| \in [0,2]$.

Consequently

$f(|x|)=|x|-1$ for all $x \in [-2,2]$

$$\Rightarrow f(|x|)=\begin{cases}-x-1, \text{for all } x \in [-2,0]\\ x-1, \text{for all } x \in [0,2]\end{cases} \quad \dots(i)$$

$$\text{Now, } f(|x|)=\begin{cases}-1, & -2 \leq x < 0\\ x-1, & 0 \leq x \leq 2\end{cases}$$

$$\Rightarrow |f(x)|=\begin{cases}1, & -2 \leq x < 0\\ 1-x, & 0 \leq x < 1\\ x-1, & 1 \leq x \leq 2\end{cases} \quad \dots(ii)$$

From (i) and (ii), we get

$g(x)=f(|x|)+|f(x)|$

$$\Rightarrow g(x)=\begin{cases}-x-1+1, & -2 \leq x < 0\\ x-1+1-x, & 0 \leq x < 1\\ x-1+x-1, & 1 \leq x \leq 2\end{cases}$$

$$\Rightarrow g(x)=\begin{cases}-x, & -2 \leq x < 0\\ 0, & 0 \leq x < 1\\ 2(x-1), & 1 \leq x \leq 2\end{cases}$$

16 **(d)**

Given, $f(x)=x-3,\ g(x)=x^2+1$

$\therefore \quad g\{f(x)\}=g(x-3)$

$\Rightarrow \quad 10=(x-3)^2+1$

$\Rightarrow \quad 10=x^2+10-6x$

$\Rightarrow \quad x(x-6)=0 \Rightarrow x=0,6$

17 **(c)**

We have,

$g(f(x))=8$

$\Rightarrow g(2x+3)=8$

$\Rightarrow (2x+3)^2+7=8 \Rightarrow 2x+3=\pm 1 \Rightarrow x=-1,-2$

18 **(c)**

Given, $f(x)=\frac{1}{\sqrt{4-x^2}}$

For domain of $f(x)$,

$\Rightarrow \quad 4-x^2 > 0$

$\Rightarrow \quad x^2 < 4$

$\Rightarrow \quad -2 < x < 2$

$\therefore$ Domain $= (-2, 2)$

19 **(b)**

Given, $f(0) = 1, \quad f(1) = 5, \quad f(2) = 11$

Let the second degree equation be

$$f(x) = ax^2 + bx + c$$

$\therefore \quad f(0) = 0 + 0 + c \Rightarrow c = 1 \quad$ ... (i)

$f(1) = a + b + c \Rightarrow 5 = a + b + 1$

$\Rightarrow \quad a + b = 4 \quad$ ... (ii)

$f(2) = 4a + 2b + c \Rightarrow 4a + 2b + 1 = 11$

$\Rightarrow \quad 2a + b = 5 \quad$ ... (iii)

On solving Eqs. (ii) and (iii), we get

$a = 1, \quad b = 3$

$\therefore$ The required equation is

$$f(x) = x^2 + 3x + 1$$

20 **(c)**

We have,

$$[x] + \sum_{r=1}^{2000} \frac{\{x+r\}}{2000}$$

$$= [x] + \frac{1}{2000} \sum_{r=1}^{2000} ((x+r) - [x+r])$$

$$\Rightarrow [x] + \sum_{r=1}^{2000} \frac{[x+r]}{2000} = [x] + \frac{1}{2000} \sum_{r=1}^{2000} (x - [x])$$

$[\because [x+r] = [x] + r]$

$$\Rightarrow [x] + \sum_{r=1}^{2000} \frac{\{x+r\}}{2000} = [x] + \frac{2000[x]}{2000} = [x] + \{x\}$$

$$= x$$

21 **(c)**

We have,

$x \in [-2, 2] \Rightarrow |x| \in [0, 2]$

$\therefore f(|x|) = |x| - 1$

Now,

$f(|x|) = x$

$\Rightarrow |x| - 1 = x \Rightarrow -x - 1 = x$ for $x \le 0 \Rightarrow x = -\frac{1}{2}$

Hence, $\{x \in [-2, 2] : x \le 0 \text{ and } f(|x|) = x\} = \left\{-\frac{1}{2}\right\}$

22 **(a)**

Since the function $g(x) = \cos x$ is an even function and $h(x) = \log\left(x + \sqrt{x^2+1}\right)$ is an odd function

Therefore, the function $goh\ (x) = \cos\left(\log\left(x + \sqrt{x^2+1}\right)\right)$ is an even function

23 **(a)**

Given

$f(\theta) = 4 + 4\sin^3\theta - 3\sin\theta$

$\quad = 4 - (3\sin\theta - 4\sin^3\theta) = 4 - \sin 3\theta$

$\therefore$ Period of $f(\theta) = \frac{2\pi}{3}$

24 **(d)**

Given, $f(2x+3) = \sin x + 2^x$

Put $\quad x = 2m - n$

$\therefore f[2(2m-n)+3] = \sin(2m-n) + 2^{2m-n}$

$\Rightarrow \quad f(4m - 2n + 3) = \sin(2m-n) + 2^{2m-n}$

25 **(b)**

We have, $f(x) = \dfrac{x+2}{x^2 - 8x - 4}$

For $f(x)$ to be defined, we must have

$x^2 - 8x - 4 \neq 0$, i.e., $x \neq 4 \pm 2\sqrt{5}$

$\therefore$ Domain $(f) = R - \{4 - 2\sqrt{5}, 4 + 2\sqrt{5}\}$

Let $y = f(x)$. Then,

$$y = \frac{x+2}{x^2 - 8x - 4}$$

$$\Rightarrow x^2 y - (8y+1)x - (4y+2) = 0$$

$$\Rightarrow x = \frac{(8y+1) \pm \sqrt{(8y+1)^2 + 4y(4y+2)}}{2y}$$

$$\Rightarrow x = \frac{(8y+1) \pm \sqrt{80y^2 + 24y + 1}}{2y}$$

For x to be real, we must have

$80y^2 + 24y + 1 \ge 0$ and $y \neq 0$

$\Rightarrow (20y+1)(4y+1) \ge 0$ and $y \neq 0$

$\Rightarrow y \le -\frac{1}{4}$ or, $y \ge -\frac{1}{20}, y \neq 0$

$\Rightarrow y \in (-\infty, -1/4] \cup [-1/20, \infty)$ and $y \neq 0$

For $x = -2$, we have $y = 0$ and $-2 \in$ Domain (f)

Hence, range $(f) = (-\infty, -1/4] \cup [-1/20, \infty)$

26 **(d)**

Since $f(x)$ is a periodic function with period $2\pi/5$. Therefore, f is not injective. The function f is not surjective also as its range $[-1,1]$ is a proper subset of its co-domain R

27 **(b)**

It is clear from the given options that $\cos\sqrt{x} + \cos^2 x$ is not periodic.

28 **(b)**

Given, $f(x) = [2x] - 2[x], \forall x \in R$

If x is an integer, then

$f(x) = 0$

And if x is an integer, then

$f(x)$ is either 1 or 0.

$\therefore$ Range of $f(x) = \{0, 1\}$

29 **(b)**

Since, $\quad g(f(x)) = |\sin x|$

$\Rightarrow \quad g(\sin^2 x) = |\sin x|$

$\Rightarrow \quad g(\sin^2 x) = \sqrt{\sin^2 x} \quad \therefore \quad g(x) = \sqrt{x}$

30 **(b)**

We have,

$f:[2,\infty) \to B$ such that $f(x) = x^2 - 4x + 5$

Since f is a bijection. Therefore, B = range of f.

Also, $f(x) = x^2 - 4x + 5 = (x-2)^2 + 1$ for all $x \in [2,\infty)$

Therefore, $f(x) \geq 1$ for all $x \in [2,\infty)$. Hence, $B = [1,\infty)$

31 **(a)**

$f(x)$ is defined, if

$-(\log_2 x)^2 + 5(\log_2 x) - 6 > 0$ and $x > 0$

$\Rightarrow (\log_2 x)^2 - 5(\log_2 x) + 6 < 0$ and $x > 0$

$\Rightarrow (\log_2 x - 2)(\log_2 x - 3) < 0$ and $x > 0$

$\Rightarrow 2 < \log_2 x < 3$ and $x > 0$

$\Rightarrow 2^2 < x < 2^3$ and $x > 0 \Rightarrow x \in (4,8)$

32 **(c)**

$$f(x+10\pi) = \sin\left\{\sin\left(\frac{x+10\pi}{5}\right)\right\}$$

$$\Rightarrow f(x+10\pi) = \sin\left\{\sin\left(\frac{x}{5}+2\pi\right)\right\}$$

$$\Rightarrow f(x+10\pi) = \sin\left\{\sin\left(\frac{x}{5}\right)\right\} = f(x)$$

Therefore, period of $f(x)$ is 10π.

33 **(a)**

The function $f(x) = \sqrt{\log_{10}\left(\frac{5x-x^2}{4}\right)}$ is defined, if

$\frac{5x - x^2}{4} \geq 1 \Rightarrow 5x - x^2 - 4 \geq 0 \Rightarrow x \in [1,4]$

$\therefore$ Domain (f) $= [1,4]$

34 **(b)**

Since, $-1 \leq \cos 3x \leq 1$

$\Rightarrow 1 \leq -\cos 3x \leq -1$

$\Rightarrow 3 \leq 2 - \cos 3x \leq 1$

$\Rightarrow \frac{1}{3} \leq \frac{1}{2-\cos 3x} \leq 1$

$\therefore$ Range of f is $\left[\frac{1}{3}, 1\right]$.

35 **(c)**

We have,

$$f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 1-x, & \text{if } x \text{ is irrational} \end{cases}$$

If x is rational, then $f(x) = x$

$\therefore f(f(x)) = f(x) = x$

If x is irrational, then $f(x) = 1 - x$

$\therefore f(f(x)) = f(1-x) = 1-(1-x) = x$

Thus, $f(f(x)) = x$ for all $x \in [0,1]$

36 **(c)**

Let $y = \frac{x}{1+x^2}$

$\Rightarrow x^2 y - x + y = 0$

For x to be real

$1 - 4y^2 \geq 0$

$\Rightarrow (1-2y)(1+2y) \geq 0$

$\Rightarrow \left(\frac{1}{2} - y\right)\left(\frac{1}{2} + y\right) \geq 0$

$\Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$

$\therefore y = f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

37 **(d)**

The domain of $f(x)$ is the complete set of real numbers. Since $f: R \to A$ is a surjection.

Therefore, A is the range of $f(x)$

Let $f(x) = y$. Then, $y \geq 0$

Now,

$f(x) = y$

$\Rightarrow \frac{x^2}{x^2+1} = y$

$\Rightarrow \frac{x^2+1}{x^2} = \frac{1}{y}$ for $y > 0$

$\Rightarrow \frac{1}{x^2} = \frac{1-y}{y} \Rightarrow x = \sqrt{\frac{y}{1-y}}$

Now,

$x \in R, \Rightarrow \sqrt{\frac{y}{1-y}}$ is real $\Rightarrow \frac{y}{1-y} \geq 0 \Rightarrow 0 \leq y < 1$

Therefore, range of $f(x)$ is $[0,1)$. Hence, $A = [0,1)$

38 **(c)**

Since, inverse of an equivalent relation is also an equivalent relation.

$\therefore R^{-1}$ is an equivalent relation.

39 **(d)**

The domain of $f(x)$ is the complete set of real numbers. Since $f: R \to A$ is a surjection.

Therefore, A is the range of $f(x)$

Let $f(x) = y$. Then, $y \geq 0$ and, $f(x) = y$

$\therefore \frac{x^2}{x^2+1} = y$

$\Rightarrow \frac{x^2+1}{x^2} = \frac{1}{y}$ for $y > 0$

$\Rightarrow \frac{1}{x^2} = \frac{1-y}{y} \Rightarrow x = \sqrt{\frac{y}{1-y}}$

Now,

$\sqrt{\frac{y}{1-y}}$ is real $\Rightarrow \frac{y}{1-y} \geq 0 \Rightarrow 0 \leq y < 1$

So, Range of $f(x)$ is $[0,1)$. Hence, $A = [0,1)$

40 **(c)**

For $f(x)$ to be defined, $5 - 4x - x^2 \geq 0$ and $x + 4 > 0$

$\Rightarrow -5 \leq x \leq 1$

And $x > -4$

$\Rightarrow -4 < x \leq 1$

41 **(d)**

$\because\ f(x) = a^{\{\tan(\pi x)+x-[x]\}}$

$= a^{\{\tan(\pi x)+(x)\}}$

$= a^{\tan \pi x} a^{\{x\}}$

Hence, period of $f(x)$ is 1.

42 **(a)**

For $f(x)$ to be defined

$x - 1 > 0$ and $2x - 1 > 0$ and $2x - 1 \neq 1$

$\Rightarrow x > 1, x > \frac{1}{2}$ and $x \neq 1$

$\Rightarrow x > 1$

Hence, domain is $(1, \infty)$.

43 **(c)**

We have,

$f(x) = \sin x$ and $g(x) = x^2$

$\therefore fog(x) = f(g(x)) = f(x^2) = \sin x^2$

44 **(d)**

$f(x) = f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right)\right] + f(x\,y)$

$= \cos\ (\log x). \cos\ (\log y)$

$-\frac{1}{2}\left[\cos\left(\log\left(\frac{x}{y}\right)\right) + \cos\ (\log xy)\right]$

$= \cos\ (\log x) \cos\ (\log y) - \frac{1}{2}$

$\times 2 \cos\ (\log x) \cos\ (\log y)$

$= \cos\ (\log x) \cos\ (\log y)$

$- \cos\ (\log x) \cos\ (\log y)$

$= 0$

45 **(b)**

We have,

$f(x) = \sqrt{\frac{-\log_{0.3}(x-1)}{-x^2+3x+18}} = \sqrt{\frac{\log_{0.3}(x-1)}{x^2-3x-18}}$

$f(x)$ is defined, if

$\frac{\log_{0.3}(x-1)}{x^2-3x-18} \geq 0$

$\Rightarrow \begin{cases} \log_{0.3}(x-1) \geq 0 \text{ and } x^2 - 3x - 18 > 0 \\ \text{OR} \\ \log_{0.3}(x-1) < 0 \text{ and } x^2 - 3x - 18 < 0 \end{cases}$

$\Rightarrow \begin{cases} 1 < x \leq 2 \text{ and } x < -3 \text{ or } x > 6 \\ \text{OR} \\ x > 2 \text{ and } -3 < x < \end{cases}$

$\Rightarrow 2 < x < 6 \Rightarrow x \in (2,6)$

Hence domain of $f(x) = (2, 6)$

46 **(c)**

For even $f(-x) = f(x)$ and for odd, $f(-x) = -f(x)$

And $f(x)$ is increasing, if $f'(x) > 0$.

Here, $f(x)$ is not differentiable at $x \in I$ and above two cases are also not satisfied by $f(x)$.

$\therefore\ f(x) = [x]$ is neither even nor odd.

47 **(a)**

For $f(x)$ to be real, we must have

$-\log_4\left(\frac{6x-4}{6x+5}\right) > 0, \frac{6x-4}{6x+5} > 0$ and $6x + 5 \neq 0$

$\Rightarrow \log_4\left(\frac{6x-4}{6x+5}\right) < 0, \frac{6x-4}{6x+5} > 0$ and $6x + 5 \neq 0$

$\Rightarrow \frac{6x-4}{6x+5} > 4^0, \frac{6x-4}{6x+5} > 0$ and $x \neq \frac{-5}{6}$

$\Rightarrow \frac{-9}{6x+5} < 0, \frac{6x-4}{6x+5} > 0$ and $x \neq \frac{-5}{6}$

$\Rightarrow 6x + 5 > 0, \frac{6x-4}{6x+5} > 0$ and $x \neq \frac{-5}{6}$

$\Rightarrow 6x - 4 > 0$ and $x \neq \frac{-5}{6}$

$\Rightarrow x > \frac{2}{3}$ and $x \neq \frac{-5}{6}$

$\Rightarrow x \in (2/3, \infty)$

48 **(c)**

R is not anti-symmetric.

49 **(d)**

Given, $n(A) = 4$ and $n(B) = 6$

Here, $n(B) > n(A)$

Since, the function f is one-one and onto.

$\therefore$ Required number of ways

$= {}^6P_4 = \frac{6!}{2!} = 360$

50 **(d)**

We have,

$f(x^2) = x^2 - \frac{1}{x^2} = \left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)$

$= \left(x + \frac{1}{x}\right) f(x)$

51 **(d)**

We have,

$f(x^2) = |x^2 - 1| \neq |x - 1|^2 = [f(x)]^2$

$f(|x|) = ||x| - 1 \neq |x - 1| = |f(x)|$

and,

$f(x + y) = |x + y - 1| \neq |x - 1| + |y - 1|$

$\neq f(x) + f(y)$

Hence, none of the given option is true

52 **(d)**

Given,

$f(x + y) = f(x) + f(y)$

For $x = 1,\ y = 1$ we get

$f(2) = f(1) + f(1)$

$= 2. f(1) = 10$

Also

$f(3) = f(2) + f(1) = 15$

$\Rightarrow\ f(n) = 5n$

$\therefore\ f(100) = 500$

53 **(d)**

Since, R is defined as aRb iff $|a-b|>0$.
Reflexive : aRa iff $|a-a|>0$
Which is not true. So, R is not reflexive.
Symmetric : aRb iff $|a-b|>0$
Now, bRa iff $|b-a|>0$
$\Rightarrow \quad |a-b|>0 \quad \Rightarrow \quad aRb$
Thus, R is symmetric.
Transitive : aRb iff $|a-b|>0$
bRc iff $|b-c|>0$
$\Rightarrow \quad |a-b+b-c|>0$
$\Rightarrow \quad |a-c|>0$
$\Rightarrow \quad |c-a|>0 \Rightarrow aRc$
Thus, R is also transitive.

54 **(a)**

$$fof=\frac{1-f(x)}{1+f(x)}=\frac{1-\frac{1-x}{1+x}}{1+\frac{1-x}{1+x}}$$

$\Rightarrow \quad f[f(x)]=x$
$\Rightarrow \quad f(x)=f^{-1}(x)$

55 **(b)**

Given, $f(x)=\log\left(x+\sqrt{x^2+1}\right)$
$\therefore \ f(x)+f(-x)$

$$=\log(x+\sqrt{x^2+1}$$
$$+\log(-x+\sqrt{x^2+1})$$
$$=\log(1)=0$$

Hence, $f(x)$ is an odd function.

56 **(c)**

Given,
$f(x)=\log\{(ax^2+bx+c)(x+1)\}$
$=\log(ax^2+bx+c)+\log(x+1)$
For $f(x)$ to be defined
$ax^2+bx+c>0$ and $x+1>0$
$\Rightarrow \quad x>-1$
Hence, option (c) is correct.

57 **(d)**

We have, $f(x)=x^2+x$
Clearly, $y=x^2+x$ is a parabola opening upward having its vertex at $\left(-\frac{1}{2},-\frac{1}{4}\right)$. So, f is a many-one into function
ALITER We have, $f(0)=f(-1)=0$
So, f is many-one
Also, $f(x)=x^2+x=\left(x+\frac{1}{2}\right)^2-\frac{1}{4}\geq-\frac{1}{4}$ for all x
$\therefore$ Range $(f)=[-1/4,\infty]\neq$ Co-domain (f)
So, f is into

58 **(c)**

We have,

$$T_1=1 \text{ and } T_2=\frac{1}{3}$$

Clearly, $T_1=3\,T_2$

59 **(c)**

Let $x+y=u$ and $x-y=v$
$\Rightarrow \quad x=\frac{u+v}{2}$ and $y=\frac{u-v}{2}$

$$\therefore \quad f(u,v)=\left(\frac{u+v}{2}\right)\left(\frac{u-v}{2}\right)$$

The arithmetic mean of $f(u,v)$ and $f(v,u)$

$$=\frac{f(u,v)+f(v,u)}{2}$$
$$=\frac{\frac{u+v}{2}\left(\frac{u-v}{2}\right)+\left(\frac{u+v}{2}\right)\left(\frac{v-u}{2}\right)}{2}=0$$

60 **(c)**

Since, $f(x)=x-[x]-\frac{1}{2}$
Also, $f(x)=\frac{1}{2}$

$$\therefore \quad \frac{1}{2}=x-[x]-\frac{1}{2}$$

$\Rightarrow \ x-[x]=1$
$\Rightarrow \ \{x\}=1 \qquad [\because x=[x]+\{x\}]$
Which is not possible.
$\therefore \left\{x\in R: f(x)=\frac{1}{2}\right\}$ is an empty set.

61 **(b)**

We know that $|\sin x|+|\cos x|$ is periodic with period $\frac{\pi}{2}$
$\therefore f(x)=|\sin 3x|+|\cos 3x|$ I periodic with period $\frac{\pi}{6}$

62 **(d)**

Given,
$f(x)=x-[x]$
For $2<x<3$, then value of $[x]$ is 2
Let $\quad y=f(x)=x-2, 2<x<3$
$\Rightarrow \quad x=2+y$
$\therefore \quad f^{-1}(x)=2+x$

63 **(a)**

We have,

$$f(x)=\left(\frac{1}{2}\right)^{\sin x}$$

Since, $\sin x$ is a periodic function with period 2π. Therefore, $f(x)$ is periodic with period 2π. We also know that every function can be uniquely expressed as the sum of an even function and an odd function
Hence, option (a) is true.

64 **(d)**

Given, $f(x)=\sqrt{x}$

$$\therefore \quad \frac{f(25)}{f(16)+f(1)}=\frac{\sqrt{25}}{\sqrt{16}+\sqrt{1}}$$
$$=\frac{5}{4+1}=1$$

66 **(a)**

The even extension of $f(x)$ on the interval $[-1,1]$ is given by

$g(x)=\begin{cases} f(x) & \text{for } 0\le x\le 1 \\ f(-x) & \text{for } -1\le x<0 \end{cases}$

$\Rightarrow g(x)$

$=\begin{cases} 3x^2-4x+8\log(1+|x|) & \text{for } 0\le x\le 1 \\ 3x^2+4x+8+\log(1+|x|) & \text{for } -1\le x<0 \end{cases}$

67 **(c)**

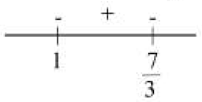
Let $y=\frac{x^2+x+2}{x^2+x+1}$

$\Rightarrow x^2(y-1)+x(y-1)+(y-2)=0, \forall\, x\in R$

Now, $D\ge 0 \Rightarrow (y-1)^2-4(y-1)(y-2)\ge 0$

$\Rightarrow (y-1)\{(y-1)-4(y-2)\}\ge 0$

$\Rightarrow (y-1)(-3y+7)\ge 0$



$\Rightarrow 1\le y\le \frac{7}{3}$

68 **(a)**

We observe that

Period of $\sin\left(\frac{\pi x}{2}\right)$ is $\frac{2\pi}{\pi/2}=4$, Period of $\cos\frac{\pi x}{2}$ is $\frac{2\pi}{\pi/2}=4$,

So, period of $\sin\frac{\pi x}{2}+\cos\frac{\pi x}{2}$ is LCM of $(4,4)=4$

69 **(b)**

We have,

$f(x)=\sin^4 x+\cos^4 x$

$\Rightarrow f(x)=(\sin^2 x+\cos^2 x)-2\sin^2 x\cos^2 x$

$\Rightarrow f(x)=1-\frac{1}{2}(\sin 2x)^2=1-\frac{1}{2}\left\{\frac{1-\cos 4x}{2}\right\}$

$=\frac{3}{4}+\frac{1}{4}\cos 4x$

Since $\cos x$ is periodic with period 2π. Therefore, $\cos 4x$ is periodic with period $\pi/2$ and hence $f(x)$ is periodic with period $\pi/2$

70 **(a)**

Given, $(x,y)\Leftrightarrow x^2-4xy+3y^2=0$

Or $(x,y)\Leftrightarrow (x-y)(x-3y)=0$

(i) **Reflexive**

$xRx \Rightarrow (x-x)(x-3x)=0$

$\therefore$ It is reflexive.

(ii) **Symmetric**

Now, $xRy \Leftrightarrow (x-y)(x-3y)=0$

And, $yRx \Leftrightarrow (y-x)(y-3x)=0 \Rightarrow xRy\ne yRx$

$\therefore$ It is not symmetric.

Similarly, it is not transitive.

71 **(b)**

We have,

$f(x)=(x-1)(x-2)(x-3)$

$\Rightarrow f(1)=f(2)=f(3)=0$

$\Rightarrow f(x)$ is not one-one

For each $y\in R$, there exists $x\in R$ such that $f(x)=y$. Therefore, f is onto

Hence, $f:R\to R$ is onto but not one-one

72 **(c)**

Since, $f:X\to Y$ and $f(x)=\sin x$

Now, take option (c).

Domain $=\left[0,\frac{\pi}{2}\right]$, Range $=[-1,1]$

For every value of x, we get unique value of y. But the value of y in $[-1,0)$ does not have any preimage.

$\therefore$ Function is one-one but not onto.

73 **(c)**

Since, $f:R\to R$ such that $f(x)=3^{-x}$

Let y_1 and y_2 be two elements of $f(x)$ such that $y_1=y_2$

$\Rightarrow 3^{-x_1}=3^{-x_2} \Rightarrow x_1=x_2$

Since, if two images are equal, then their elements are equal, therefore it is one-one function.

Since, $f(x)$ is positive for every value of x, therefore $f(x)$ in into.

On differentiating w.r.t. x, we get $\frac{dy}{dx}=-3^{-x}\log 3<0$ for every value of x.

$\therefore$ It is decreasing function.

$\therefore$ Statement I and II are true.

74 **(d)**

We have,

$f(x)=x[x]=kx$, when $k\le x<k+1$ and $k\in Z$

Clearly, it is not a periodic function

75 **(a)**

Let $f(x)=y$. Then,

$\frac{3x+2}{5x-3}=y \Rightarrow x=\frac{3y+2}{5y-3}$

$\therefore f^{-1}(y)=\frac{3y+2}{5y-3}$ or, $f^{-1}(x)=\frac{3x+2}{5x-3}$

$=f(x)$ for all x

76 **(c)**

Given, $f(x)=\frac{\sqrt{4-x^2}}{\sin^{-1}(2-x)}$

For $f(x)$ to be defined $4-x^2\ge 0$; $-1\le 2-x\le$ and $2-x\ne 0$

$\Rightarrow -2\le x\le 2$; $1\le x\le 3$ and $x\ne 2$

$\therefore$ Domian of $f(x)$ is $[1,2)$.

77 **(a)**

Clearly, $f(x)$ is defined for all x satisfying

$-1\le |x-1|-2\le 1$

$\Rightarrow 1\le |x-1|\le 3$

$\Rightarrow 1\le (x-1)\le 3$ or, $-3\le x-1\le -1$

$\Rightarrow 2 \le x \le 4$ or, $-2 \le x \le 0 \Rightarrow x \in [2,4] \cup [-2,0]$

78 **(a)**

For $f(x)$ to be defined, we must have

$-1 \le [\sec x] \le 1$

$\Rightarrow -1 \le \sec x < 2$

$\Rightarrow 2m\pi \le x < 2m\pi + \frac{\pi}{3}, m \in Z$ or, $x = (2n+1)\pi, n \in Z$

$\Rightarrow x \in \{x : x = (2n+1)\pi, n \in Z\}$
$\cup \{x : 2m\pi \le x < 2m\pi + \pi/3, m \in Z\}$

79 **(d)**

For domain of $\sin^{-1}(\log_3 x)$

$-1 \le \log_3 x \le 1$

$\Rightarrow \quad 3^{-1} \le x \le 3$

$\therefore$ Domain of $\sin^{-1}(\log_3 x)$ is $\left[\frac{1}{3}, 3\right]$.

80 **(b)**

We have,

$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$

$\Rightarrow f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^2 - 2$

$\Rightarrow f(y) = y^2 - 2$, where $y = x + \frac{1}{x}$

Now,

$y = x + \frac{1}{x}, x \ne 0$

$\Rightarrow y \ge 2$ or, $y \le -2 \Rightarrow |y| \ge 2$

Thus, $f(y) = y^2 - 2$ for all $|y| \ge 2$

81 **(a)**

Given, $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$

$$= \sin^2 x + \left[\sin x \cos\frac{\pi}{3} + \cos x \sin\frac{\pi}{3}\right]^2 + \cos x\left[\cos x \cos\frac{\pi}{3} - \sin x \sin\frac{\pi}{3}\right]$$

$$= \sin^2 x + \left[\frac{\sin x}{2} + \cos x . \frac{\sqrt{3}}{2}\right] + \cos x\left[\frac{\cos x}{2} - \sin x . \frac{\sqrt{3}}{2}\right]$$

$$= \sin^2 x + \frac{\sin^2 x}{4} + \frac{3\cos^2 x}{4} + \sin x \cos x . \frac{\sqrt{3}}{2} + \frac{\cos^2 x}{2} - \sin x \cos\frac{\sqrt{3}}{2}$$

$$= \frac{5\sin^2 x}{4} + 5\frac{\cos^2 x}{4} = \frac{5}{4}$$

$\therefore \; gof(x) = g[f(x)] = g\left(\frac{5}{4}\right) = 1$ (given)

82 **(a)**

We have,

$f(x) = \sec\left(\frac{\pi}{4}\cos^2 x\right), x \in (-\infty, \infty)$

Clearly,

$0 \le \frac{\pi}{4}\cos^2 x \le \frac{\pi}{4}$ for all $x \in (-\infty, \infty) \Rightarrow f(x) \in [1, \sqrt{2}]$

83 **(d)**

We have,

$$f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}} = \begin{cases} \frac{e^x - e^{-x}}{e^x + e^{-x}}, & x \ge 0 \\ 0, & x < 0 \end{cases}$$

$\Rightarrow f(x)$ is many-one into as range $(f) = [0, \infty)$

84 **(b)**

Given, $\quad f(x) = (x-1)(x-2)(x-3)$

$\Rightarrow \quad f(1) = f(2) = f(3) = 0$

$\Rightarrow f(x)$ is not one-one.

For each $y \in R$, there exists $x \in R$ such that $f(x) = y$.

Therefore, f is onto.

85 **(c)**

Given, $f(n) = \begin{cases} \frac{n-1}{2}, \text{ when } n \text{ is odd} \\ -\frac{n}{2}, \text{ when } n \text{ is even} \end{cases}$

And $f: N \to I$, where N is the set of natural numbers and I is the set of integers.

Let $x, y \in N$ and both are even.

Then, $f(x) = f(y)$

$\Rightarrow -\frac{x}{2} = -\frac{y}{2} \Rightarrow x = y$

Again, $x, y \in N$ and both are odd.

Then, $f(x) = f(y)$

$\Rightarrow \quad \frac{x-1}{2} = \frac{y-1}{2}$

$\Rightarrow \quad x = y$

So, mapping is one-one.

Since, each negative integer is an image of even natural number and positive integer is an image of odd natural number. So, mapping is onto.

86 **(a)**

Since $\sqrt{\cos(\sin x)}$ exists for all $x \in R$ and $\sin^{-1}\left(\frac{1+x^2}{2x}\right)$ exists for $x = \pm 1$. Therefore,

$f(x) = \sqrt{\cos(\sin x)} + \sin^{-1}\left(\frac{1+x^2}{2x}\right)$ is defined for $x \in [-1, 1]$

87 **(a)**

Here, $f(x)=\log\frac{10+x}{10-x}$

Given that, $f(x)=k\,f\left(\frac{200x}{100+x^2}\right)$

$\Rightarrow \log\frac{10+x}{10-x}=k.\log\left\{\frac{10+\frac{200x}{100+x^2}}{10-\frac{200x}{100+x^2}}\right\}$

$=k\log\left(\frac{10+x}{10-x}\right)^2$

$\Rightarrow \log\frac{10+x}{10-x}=2k\log\frac{10+x}{10-x}$

$\Rightarrow \quad k=0.5$

88 **(d)**

Since, $f(n)=\begin{cases} n^2, & \text{if } n \text{ odd} \\ 2n+1, & \text{if } n \text{ is even}\end{cases}$

$f(1)=1^2=1 \quad f(2)=2(2)+1=5$

$f(3)=3^2=9 \quad f(4)=2(4)+1=9$

$\therefore \quad f(3)=f(4)$

$\therefore f$ is not injective.

Also, f is not surjective as every element of N is not the image of any element of N

89 **(a)**

$\because f(y)=f\left(\frac{x+2}{x-1}\right)=\frac{\frac{x+2}{x-1}+2}{\frac{x+2}{x-1}-1}$

$\therefore \quad f(y)=x$

90 **(c)**

$(fog)(x)=f[g(x)]=f(|3x+4|)$

Since, the domain of f is $[-3,5]$

$\therefore \quad -3\le|3x+4|\le 5$

$\Rightarrow \quad |3x+4|\le 5$

$\Rightarrow \quad -5\le 3x+4\le 5$

$\Rightarrow \quad -9\le 3x\le 1$

$\Rightarrow \quad -3\le x\le\frac{1}{3}$

$\therefore$ Domian of fog is $\left[-3,\frac{1}{3}\right]$.

92 **(b)**

$g(x)=1+x-[x]$ is greater than 1 since $x-[x]>0$,

$f\{g(x)\}=1$

93 **(c)**

We have,

$f(x)=x-[x]$

$\Rightarrow f(x)=\begin{cases} x-n, & \text{if } n<x<n+1 \\ n-n=0, & \text{if } x=n\end{cases}$, where $n\in Z$

Thus, $f(x)$ is a many-one function

Consequently, $f^{-1}(x)$ is not defined

94 **(a)**

Given, $P(x)=x+ax+b$

$\therefore \quad P(10)=10+10a+b=10+5=15$

And $\quad P(11)=11+11a+b$

$=11+5+a=16+a$

$\because \quad P(10)P(11)=P(n)$

$\Rightarrow \quad 15(16+a)=n+na+b$

$\Rightarrow \quad 240+15a=n+na+5-10a$

$\Rightarrow \quad n+na-25a-235=0$

(a) When $n=15$

$15+15a-25a-235=0$

$\Rightarrow \quad a=-22$ and $b=225$

(b) When $n=64$

$65+65a-25a-235=0$

$\Rightarrow \quad a=-\frac{17}{4}$ which is not integer.

(c) When $n=115$

$115+115a-25a-235=0$

$\Rightarrow \quad a=\frac{4}{3}$ which is not integer.

(d) When $n=165$

$165+165a-25a-235=0$

$\Rightarrow \quad a=\frac{1}{2}$ which is not integer.

96 **(a)**

We have,

$fof^{-1}(x)=x$ for all $x\in(-\infty,2]$

$\Rightarrow f\left(f^{-1}(x)\right)=x$

$\Rightarrow 4f^{-1}(x)-\{f^{-1}(x)\}^2=x$

$\Rightarrow \{f^{-1}(x)\}^2-4f^{-1}(x)+x=0$

$\Rightarrow f^{-1}(x)=\frac{4\pm\sqrt{16-4x}}{2}=2\pm\sqrt{4-x}$

$\Rightarrow f^{-1}(x)=2-\sqrt{4-x} \qquad [\because -\infty<f^{-1}(x)\le 2]$

97 **(b)**

We have,

$f(x)=(a-x^n)^{1/n}, n\in N$

$\Rightarrow fof(x)=f\left(f(x)\right)$

$\Rightarrow fof(x)=f\left((a-x^n)^{1/n}\right)$

$\Rightarrow fof(c)=\left[a-\left\{(a-x^n)^{1/n}\right\}^n\right]^{1/n}$

$\Rightarrow fof(x)=\{a-(a-x^n)\}^{1/n}=(x^n)^{1/n}=x$

98 **(d)**

We have, $f(x)=\log_3|\log_e x|$

Clearly, $f(x)$ is defined, if

$\log_e x\ne 0$ and $x>0\Rightarrow x\ne 1$ and $x>0\Rightarrow x\in(0,1)\cup(1,\infty)$

99 **(d)**

Since $f:R\to R$ and $g:R\to R$, given by $f(x)=2x-3$ and $g(x)=x^3+5$ respectively, are bijections. Therefore, f^{-1} and g^{-1} exist

We have,

$f(x)=2x-3$

$\therefore f(x)=y$

$\Rightarrow 2x-3=y\Rightarrow x=\frac{y+3}{2}$

$\Rightarrow f^{-1}(y)=\frac{y+3}{2}$

Thus, f^{-1} is given by $f^{-1}(x)=\frac{x+3}{3}$ for all $x\in R$

Similarly, $g^{-1}(x)=(x-5)^{1/3}$ for all $x\in R$

Now, $(fog)^{-1}(x)=(g^{-1}of^{-1})(x)=g^{-1}(f^{-1}(x))$

$$\Rightarrow (fog)^{-1}(x)=g^{-1}\left(\frac{x+3}{2}\right)=\left(\frac{x+3}{2}-5\right)^{1/3}$$

$$=\left(\frac{x-7}{2}\right)^{1/3}$$

100 **(c)**

Since, $f(x)$ is a many-one function. so its inverse does not exist.

101 **(b)**

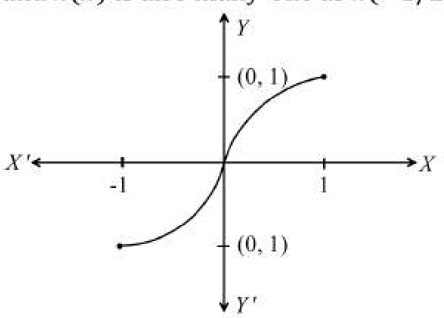
Clearly, $f(x)=\frac{x}{2}$ is one-one but not onto as range of f is $[1/2,1/2]\neq A$

The graph of $g(x)=\sin\left(\frac{\pi x}{2}\right)$ is as shown in Fig.S.1

Evidently, it is a bijection

$h(x)=|x|$ is many one as $h(-1/2)=h(1/2)$

and $k(x)$ is also many-one as $k(-1/2)=k(1/2)$



102 **(b)**

For domain of $f(x)$, $2-2x-x^2\geq 0$

$\Rightarrow \quad x^2+2x-2\leq 0$

$\Rightarrow \quad -1-\sqrt{3}\leq x<-1+\sqrt{3}$

103 **(c)**

$$fogoh\,(x)=(fog)(h(x))=(fog)(2x)$$

$$=f(g(2x))=f([4x^2])$$

$$\Rightarrow fogoh\,(x)=\begin{cases} f(1), & \text{if}\ \frac{1}{2}\leq x<\frac{1}{\sqrt{2}} \\ f(2), & \text{if}\ x=\frac{1}{\sqrt{2}}\end{cases}$$

$$\Rightarrow fogoh\,(x)=\begin{cases} \sin^{-1}(1), & \text{if}\ \frac{1}{2}\leq x<\frac{1}{\sqrt{2}} \\ \sin^{-1}(2), & \text{if}\ x=\frac{1}{\sqrt{2}}\end{cases}$$

$$\Rightarrow fogoh\,(x)=\begin{cases} \frac{\pi}{2}, & \text{if}\ \frac{1}{2}\leq x<\frac{1}{\sqrt{2}} \\ \text{Does not exist}, & \text{if}\ x=\frac{1}{\sqrt{2}}\end{cases}$$

Thus, option (a) and (b) are not correct

Now,

$hofog\,(x)=2\sin^{-1}[x^2]$ and, $hogof\,(x)=2[\{\sin^{-1}x\}^2]$

$\Rightarrow hofog\,(x)=2\sin^{-1}0$

and

$hogof\,(x)$

$$=2\times 0\quad\left[\begin{array}{c}\because \frac{1}{4}\leq x^2\leq 1/2\Rightarrow[x^2]=0 \\ \text{and} \\ \frac{1}{2}\leq x\leq\frac{1}{\sqrt{2}} \\ \Rightarrow \pi/6\leq\sin^{-1}x\leq\pi/4 \\ \Rightarrow[\{\sin^{-1}x\}^2]=\end{array}\right]$$

$\Rightarrow hofog\,(x)=hogof\,(x)$ for all $x\in[1/2,1/\sqrt{2}]$

104 **(c)**

Let $x,y\in N$ be such that

$f(x)=f(y)$

$\Rightarrow x^2+x+1=y^2+y+1$

$\Rightarrow (x-y)(x+y+1)=0$

$\Rightarrow x=y \qquad [\because x+y+1\neq 0]$

$\therefore f:N\to N$ is one-one

f is not onto, because $x^2+x+1\geq 3$ for all $x\in N$

So, 1, 2 do not have their pre-image

105 **(b)**

We have,

$$f(x)=\begin{cases}0, x=0 \\ x^2\sin\left(\frac{\pi}{2x}\right), |x|<1 \\ x|x|, |x|\geq 1\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}-x^2, & x\leq -1 \\ x^2\sin\left(\frac{\pi}{2x}\right), & -1<x<0 \\ 0, & x=0 \\ x^2\sin\left(\frac{\pi}{2x}\right), & 0<x<1 \\ x^2, & x\leq 1\end{cases}$$

$$\Rightarrow f(-x)=\begin{cases}-(-x)^2, & -x\leq -1 \\ (-x)^2\sin\left(\frac{\pi}{-2x}\right), & -1<-x<0 \\ 0, & x=0 \\ (-x)^2\sin\left(\frac{\pi}{-2x}\right), & 0<-x<1 \\ (-x)^2, & -x\geq 1\end{cases}$$

$$\Rightarrow f(-x)=\begin{cases}-x^2, & x\geq 1 \\ -x^2\sin\left(\frac{\pi}{2x}\right), & 0<x<1 \\ 0, & x=0 \\ -x^2\sin\left(\frac{\pi}{2x}\right), & -1<x<0 \\ x^2, & x\leq -1\end{cases}$$

$\Rightarrow f(-x)=-f(x)$ for all x

Hence, $f(x)$ is an odd function

106 **(d)**

Here, we have to find the range of the function which $[-1/3,1]$

108 **(b)**

The function $f(x) = x^3$ is not a surjective map from Z to itself, because $2 \in Z$ does not have any pre-image in Z. The function $f(x) = x + 2$ is a bijection from Z to itself. The function $f(x) = 2x + 1$ is not a surjection from Z to itself and $f(x) = x^2 + x$ is not an injection map from Z to self

109 **(d)**

For $f(x)$ to be real, we must have

$|\cos x| + \cos x > 0$

$\Rightarrow 2\cos x > 0$ [$\because \cos x < 0 \Rightarrow |\cos x| + \cos x = 0$]

$\Rightarrow \cos x > 0$

$\Rightarrow 2n\pi - \frac{\pi}{2} < x < 2n\,\pi + \frac{\pi}{2} \Rightarrow x \in \left((4n-1)\frac{\pi}{2}, (4n+1)\frac{\pi}{2}\right)$

Hence, domain $(f) = \left((4n-1)\frac{\pi}{2}, (4n+1)\frac{\pi}{2}\right)$

110 **(d)**

We have,

$f(x) = (25 - x^4)^{1/4}$

$\therefore fof\ (x) = f(f(x)) = f((25 - x^4)^{1/4})$

$\Rightarrow fof\ (x) = \left[25 - \{(25 - x^4)^{1/4}\}^4\right]^{1/4} = \{25 - (25 - x^4)\}^{1/4}$

$\Rightarrow fof\ (x) = x$ for all x

$\therefore fof\left(\frac{1}{2}\right) = \frac{1}{2}$

<u>ALITER</u> We have,

$$f\left(f\left(\frac{1}{2}\right)\right) = f\left(\left(25 - \frac{1}{16}\right)^{\frac{1}{4}}\right)$$

$$\Rightarrow f\left(f\left(\frac{1}{2}\right)\right) = f\left(\left(\frac{399}{16}\right)^{\frac{1}{4}}\right) = \left(25 - \frac{399}{16}\right)^{\frac{1}{4}} = \frac{1}{2}$$

111 **(b)**

$f(x) = \sec\left(\text{In}\left(x + \sqrt{1 + x^2}\right)\right) = \sec(\text{odd function}) = \text{even function}$

$\because$ sec is an even function

112 **(b)**

We have, $f(x) = \sin(\log x)$

$\therefore f(xy) + f\left(\frac{x}{y}\right) - 2f(x)\cos(\log y)$

$= \sin\{\log(xy)\} + \sin\left\{\log\left(\frac{x}{y}\right)\right\} - 2\sin(\log x)\cos(\log y)$

$= \sin(\log x + \log y) + \sin(\log x - \log y) - 2\sin(\log x)\cos(\log y)$

$= 2\sin(\log x)\cos(\log y) - 2\sin(\log x)\cos(\log y) = 0$

114 **(c)**

The total number of bijections from a set containing n elements to itself is $n\,!$. Hence, required number = (106) !

115 **(c)**

We have,

$f(x) = \log_{0.5}\left\{-\log_2\left(\frac{3x-1}{3x+2}\right)\right\}$

Clearly, $f(x)$ id defined if

$-\log_2\left(\frac{3x-1}{3x+2}\right) > 0$ and $\frac{3x-1}{3x+2} > 0$

$\Rightarrow \log_2\left(\frac{3x-1}{3x+2}\right) < 0$ and $x < -\frac{2}{3}$ or $x > \frac{1}{3}$

$\Rightarrow \frac{3x-1}{3x+2} < 20$ and $x \in (-\infty, -2/3) \cup (1/3, \infty)$

$\Rightarrow \frac{-3}{3x+2} > 0$ and $x \in (-\infty, -2/3) \cup (1/3, \infty)$

$\Rightarrow x > -\frac{2}{3}$ and $x \in (-\infty, -2/3) \cup (1/3 - \infty)$

$\Rightarrow x \in (1/3, \infty)$

117 **(b)**

$f(x) = |\sin x|$ has its inverse if it is a bijection. Clearly $f(x) = |\sin x|$ is injective if its domain is $[0, \pi/2]$. Also, $f(x)$ is surjective if its co-domain is $[0, 1]$

Hence, $f(x) = |\sin x|$ is invertible if it is a function from $[0, \pi/2]$ to $[0, 1]$

118 **(b)**

We have,

$f(x) = \log\left(x + \sqrt{x^2 + 1}\right)$

$\therefore f(-x) + f(x)$

$= \log(x + \sqrt{x^2 + 1}) + \lg(-x + \sqrt{x^2 + 1})$

$\Rightarrow f(-x) + f(x) = \log(-x^2 + x^2 + 1) = \log 1 = 0$ for all x

$\Rightarrow f(-x) = -f(x)$ for all x

$\Rightarrow f(x)$ is an odd function

119 **(a)**

$aRb \Leftrightarrow a = 2^k . b$ for some integer.

Reflexive $\therefore aRb$ for $k = 0$

Symmetric $aRb \Leftrightarrow a = 2^k b$

$\Rightarrow b = 2^{-k} a \Leftrightarrow bRa$

Transitive $aRb \Leftrightarrow a = 2^{k_1} b$

$bRc \Leftrightarrow b = 2^{k_2} c$

$\Rightarrow \quad a = 2^{k_1} . 2^{k_2} c$

$\Rightarrow \quad a = 2^{k_1 + k_2} c \Leftrightarrow aRc$

$\Rightarrow aRb, bRc \Rightarrow aRc$

$\therefore R$ is an equivalent relation.

120 **(c)**

We have,

$f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) \Rightarrow f(x) = x^n + 1$

Now,

$f(10) = 1001 \Rightarrow 10^n + 1 = 1001 \Rightarrow n = 3$

$\therefore f(x) = x^3 + 1 \Rightarrow f(20) = 20^3 + 1 = 8001$

121 **(b)**

We have,

$f(x) = \frac{\sin^4 x + \cos^4 x}{x + x^2 \tan x}$

$\Rightarrow f(-x) = \frac{\sin^4 x + \cos^4 x}{-x + x^2 \tan(-x)} = -\frac{\sin^4 x + \cos^4 x}{x + x^2 \tan x}$

$= -f(x)$

So, $f(x)$ is an odd function

Obviously, $f(x)$ is not a periodic function due to the presence of x in the denominator

122 **(b)**

Since, $[b(x+1)^2 + c(x+1) + d] - [bx^2 + cx + d] - 8x + 3$

$\Rightarrow \quad (2b)x + (b + c) = 8x + 3$

$\Rightarrow \quad 2b = 8, b + c = 3 \Rightarrow b = 4, c = -1$

123 **(c)**

Let $f(x) = bx^2 + ax + c$

Since, $f(0) = 0 \Rightarrow c = 0$

And $f(1) = 0 \Rightarrow a + b = 1$

$\therefore \quad f(x) == ax + (1-a)x^2$

Also, $f'(x) > 0$ for $x \in (0, 1)$

$\Rightarrow \quad a + 2(1-a)x > 0 \quad \Rightarrow \quad a(1 - 2x) + 2x > 0$

$\Rightarrow \quad a > \frac{2x}{2x - 1} \Rightarrow \quad 0 < a < 2$

Since, $x \in (0, 1)$

$\therefore \quad f(x) = ax + (1-a)x^2; 0 < a < 2$

124 **(c)**

Put, $x = 1, -\frac{1}{2}$ in given function respectively, we get

$2f(2) + f\left(\frac{1}{2}\right) = 2 \qquad \text{... (i)}$

And $\quad 2f\left(\frac{1}{2}\right) + f(2) = -1 \qquad \text{... (ii)}$

On solving Eqs. (i) and (ii), we get $f(2) = \frac{5}{3}$

125 **(d)**

Let $\phi(x) = f(x) - g(x)$

$= \begin{cases} x, x \in Q \\ -x, x \notin Q \end{cases}$

For one-one

Take any straight line parallel to x-axis which will intersect $\phi(x)$ only at one point.

$\Rightarrow \phi(x)$ is one-one.

Foe onto

As, $\phi(x) = \begin{cases} x, x \in Q \\ -x, x \notin Q \end{cases}$, which shows

$y = x$ and $y = -x$ for irrational values $\Rightarrow y \notin$ real numbers.

$\therefore$ Range=Codomain

$\Rightarrow \quad \phi(x)$ is onto.

Thus, $f - g$ is one-one and onto.

126 **(b)**

We have,

$y = \log_2\left\{-\log_{1/2}\left(1 + \frac{1}{x^{1/4}}\right) - 1\right\}$

Clearly, y will take real values, if

$-\log_{1/2}\left(1 + \frac{1}{x^{1/4}}\right) - 1 > 0$ and $x > 0$

$\Rightarrow \log_2\left(1 + \frac{1}{x^{\frac{1}{4}}}\right) - 1 > 0$ and $x > 0$

$\Rightarrow 1 + \frac{1}{x^{1/4}} > 2$ and $x > 0$

$\Rightarrow \frac{1}{x^{1/4}} > 1$ and $x > 0 \Rightarrow x \in (0, 1)$

127 **(b)**

We observe that $\cos^{-1}\left(\frac{2-|x|}{4}\right)$ is defined, for

$-1 \le \frac{2 - |x|}{4} \le 1$

$\Leftrightarrow -6 \le -|x| \le 2 \Leftrightarrow -2 \le |x| \le 6 \Leftrightarrow |x| \le 6$

Thus, the domain o $\cos^{-1}\left(\frac{2-|x|}{4}\right)$ is $D_1 = [-6, 6]$

The domain of $\frac{1}{\log_{10}(3-x)}$ is the set of all real numbers for which $3 - x > 0$ and $3 - x \ne 1$, i.e., $x > 3$ and $x \ne 2$

Hence, the domain of the given function is

$\{x : -6 \le x \le 6\} \cap \{x : x \ne 2, x < 3\}$

$= [-6, 2) \cup (2, 3)$

128 **(b)**

We have,

$f(x) = 1 + \frac{\sin x}{1 - \sin^2 x} = 1 + \frac{\sin x}{\cos^2 x}$

$= 1 + \tan x \sec x$

$\therefore f'(x) = \sec^3 x + \sec x \tan^2 x > 0$ for all $x \in (-\pi/2, \pi/2)$

$\Rightarrow f(x)$ is an increasing function on $(-\pi/2, \pi/2)$

Now,

$\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2}\left(1 + \frac{\sin x}{1 - \sin^2 x}\right) = \infty$

and,

$\lim_{x \to -\pi/2} f(x) = \lim_{x \to -\pi/2}\left(1 + \frac{\sin x}{1 - \sin^2 x}\right) = -\infty$

Hence, range $(f) = \left(f(-\pi/2), f(\pi/2)\right) =$ $(-\infty, \infty) = R$

129 **(a)**

If A and B are two sets having m and n elements respectively such that $1 \le n \le m$, then number of onto mapping from A to B

$$= \sum_{r=1}^{n} (-1)^{n-1} nC_r r^m$$

Here, $m = 100, n = 2$

$\therefore$ The number of onto mappings from A to B

$$= \sum_{r=1}^{2} (-1)^{2-r}\ {}^2C_r\ r^{100}$$

$$= (-1)^{2-1.2}C_1 \times 1^{100} + (-1)^{2-2.2}C_2 . 2^{100}$$

$$= 2^{100} - 2$$

130 **(c)**

Given, $f\{f(x)\} = x + 1$ ... (i)

$\therefore \quad f\{f(0)\} = x + 1$

$\Rightarrow \quad f\left(\frac{1}{2}\right) = 1 \qquad \left[\because\ f(0) = \frac{1}{2}\right]$

Now, put $x = \frac{1}{2}$ in Eq. (i), we get

$$f\left\{f\left(\frac{1}{2}\right)\right\} = \frac{1}{2} + 1$$

$$\Rightarrow \quad f(1) = \frac{3}{2}$$

131 **(c)**

We have,

$$f(x) = \frac{\sin 8x \cos x - \sin 6x \cos 3x}{\cos x \cos 2x - \sin 3x \sin 4x}$$

$$\Rightarrow f(x) = \frac{(\sin 9x + \sin 7x) - (\sin 9x + \sin 3x)}{(\cos 3x + \cos x) - (\cos x - \cos 7x)}$$

$$\Rightarrow f(x) = \frac{\sin 7x - \sin 3x}{\cos 3x + \cos 7x}$$

$$\Rightarrow f(x) = \frac{2 \sin 2x \cos 5x}{2 \cos 5x \cos 2x}$$

$$\Rightarrow f(x) = \tan 2x$$

Since $\tan x$ is period with period π. Therefore, $f(x) = \tan 2x$ is periodic with period $\frac{\pi}{2}$

133 **(b)**

Since $f(x)$ is an even function. So $f'(x)$ is an odd function

134 **(c)**

Since, $f(n) = 1 + n^2$

For one-to-one, $1 + n_1^2 = 1 + n_2^2$

$\Rightarrow \quad n_1^2 - n_2^2 = 0$

$\Rightarrow \quad n_1 = n_2 \qquad (\because n_1 + n_2 \ne 0)$

$\therefore f(n)$ is one-to-one.

But $f(n)$is not onto as every element of codomain is not the image of any element of domains.

Hence, $f(n)$ is one-to-one but not onto.

136 **(b)**

Given, $f(x) = (a - x^n)^{1/n} = g(x)$

$\therefore \quad fof(x) = f(f(x))$

$$= \left[a - \left\{(a - x^n)^{\frac{1}{n}}\right\}^n\right]^{1/n} = [a - (a - x^n)]^{1/n}$$

$$= x$$

137 **(b)**

Given, $r = \{(a, b) | a, b \in R$ and $a - b + \sqrt{3}$ is an irrational number$\}$

(i) **Reflexive**

$ara = a - a + \sqrt{3} = \sqrt{3}$ which is irrational number.

(ii) **Symmetric**

Now, $2r\sqrt{3} = 2 - \sqrt{3} + \sqrt{3} = 2$

Which is not an irrational.

Also, $\sqrt{3}r2 = \sqrt{3} - 2 + \sqrt{3} = 2\sqrt{3} - 2$ which is an irrational.

$2r\sqrt{3} \ne \sqrt{3}r2$

Which is not symmetric.

(iii) **Transitive**

Now, $\sqrt{3}r2$ and $2r4\sqrt{5}$, *ie*,

$\sqrt{3} - 2 + \sqrt{3} + 2 - 4\sqrt{5} + \sqrt{3}$

$= 2\sqrt{3} - 4\sqrt{5} + \sqrt{3} \ne \sqrt{3}r4\sqrt{5}$

$\therefore$ It is not transitive.

138 **(a)**

Given, $y = x - 3 \Rightarrow x - y = 3$

$\therefore \quad R = \{(11, 8), (13, 10)\}$

$\Rightarrow \quad R^{-1} = \{(8, 11), (10, 13)\}$

139 **(d)**

Let $y = x^2 - 6x - 14 \Rightarrow y = (x - 3)^2 - 23$

$\Rightarrow \quad x = \pm\sqrt{y + 23} + 3$

$\Rightarrow \quad f^{-1}(x) = \pm\sqrt{x + 23} + 3$

$\therefore \quad f^{-1}(2) = \pm\sqrt{25} + 3 = -2, 8$

It means we do not define a inverse function

$\therefore \quad f^{-1}(2) = \phi$

140 **(c)**

Clearly, $f(x) = \sqrt[3]{\frac{2x+1}{x^2-10x-11}}$ is defined for all x except

$x^2 - 10x - 11 = 0$ i.e. $x = 11, -1$

$\therefore$ Domain $(f) = R - \{-1, 11\}$

141 **(d)**

Period of $\sin\left(\frac{3x}{2}\right) = \frac{2\pi}{3/2} = \frac{4\pi}{3}$

And period of $\sin\left(\frac{2x}{3}\right) = \frac{2\pi}{2/3} = 3\pi$

$\therefore$ Period of $\sin\left(\frac{2x}{3}\right) + \sin\left(\frac{3x}{2}\right) = \frac{\text{LCM }(3\pi, 4\pi)}{\text{HCF }(1,3)}$

$= 12\pi$

142 **(d)**

Let $f(x) = e^{x^2/2}$

$\therefore \quad f(-x) = e^{(-x)^2} = e^{x2/2}$

And $\frac{f'(x)}{x}=\frac{1}{x}\left(e^{x^2/2}.\frac{2x}{2}\right)=e^{x^2/2}$

$\Rightarrow f(x)=f(-x)$

$=\frac{f'(x)}{x}$

143 **(c)**

Given, $f(n)=\begin{cases}\frac{n}{2}, n \text{ is even}\\0, \ n \text{ is odd}\end{cases}$

Here, we see that for every odd values of z, it will give zero. It means that it is a many one function. For every even values of z, we will get a set of integers $(-\infty,\infty)$. So, it is onto. Hence, it is surjective but not injective.

145 **(c)**

Let $f^{-1}(17)=x$. Then,

$f(x)=17\Rightarrow x^2+1=17\Rightarrow x\pm 4$

Let $f^{-1}(-3)=x$

Then, $f(x)=-3\Rightarrow x^2+1=-3\Rightarrow x^2=-4$

which is not possible for any real number x

147 **(a)**

We have,

$f(x)=\frac{|x|}{x}=\begin{cases}1, 0<x\le 4\\-1, -4\le x<0\end{cases}$

$\therefore$ Range $(f)=\{-1,1\}$

148 **(b)**

We have,

$f(x)=(9x+0.5)\log_{(0.5+x)}\left\{\frac{x^2+2x-3}{4x^2-4x-3}\right\}$

Clearly, $f(x)$ will assume real values, if

$0.5+x>0, 0.5+x\ne 1$ and $\frac{x^2+2x-3}{4x^2-4x-3}>0$

Clearly, $f(x)$ will assume real values, if

$0.5+x>0, 0.5+x\ne 1$ and $\frac{x^2+2x-3}{4x^2-4x-3}>0$

$\Rightarrow x>-\frac{1}{2}, x\ne\frac{1}{2}$ and $\frac{(x+3)(x-1)}{(2x-3)(2x+1)}>0$

$\Rightarrow x>-\frac{1}{2}, x\ne\frac{1}{2}, x\ne\frac{1}{2}$

and, $x\in(-\infty,-3)\cup(-1/2,1)\cup(3/2,\infty)$

$\Rightarrow x\in(-1/2,1/2)\cup(1/2,1)\cup(3/2,\infty)$

149 **(b)**

$ho(fog)(x)=hof\{g(x)\}$

$=hof\{\sqrt{(x^2+1)}\}$

$=h\{\left(\sqrt{x^2+1}\right)^2-1\}$

$=h\{x^2+1-1\}$

$=h\{x^2\}=x^2$

150 **(b)**

Number of reflexive relations of a set of 4 elements$=2^{4^2-4}$

$=2^{12}$

151 **(d)**

Clearly, g(x) is the inverse of $f(x)$ and is given by

$g(x)=\left(\frac{x^{1/3}-b}{a}\right)^{1/2}$

153 **(d)**

We have, $f(x)=\tan\left(\frac{\pi}{[x+2]}\right)$

Clearly, $f(x)$ is defined, if

$[x+2]\ne 0$ and $[x+2]\ne 2$

$\Rightarrow x+2\notin[0,1)$ and $x+2\in[2,3)$

$\Rightarrow x\in(-2,-1)$ and $x\notin[0,1)$

$\Rightarrow x\in(-\infty,-2)\cup[-1,0)\cup[1,\infty)$

Hence, domain of $f=(-\infty,-2)\cup[-1,0)\cup[1,\infty)$

154 **(b)**

Since, $A=\{x: -1\le x\le 1\}$

And $B=\{y: 1\le y\le 2\}$

Also, $y=f(x)=1+x^2$

For $x=-1, y=1+(-1)^2=2$

And for $x=1, y=1+1^2=2$

$\therefore f$ is not injective. (one-one)

Here, $\forall B$ their is a preimage.

Hence, f is surjective.

155 **(d)**

We have,

$f(x)=[x]=k$ for $k\le x<k+1$, where $k\in Z$

So, f is many-one into

157 **(c)**

$f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2$

$\therefore \quad f(x)=x^2-2$

158 **(c)**

The relation $R=\{(1,1),(2,2),(3,3)\}$ on the set $\{1,2,3\}$ is an equivalent relation.

159 **(c)**

We have,

$f(x)=ax+b, g(x)=cx+d$

$\therefore f(g(x))=g(f(x))$ for all x

$\Leftrightarrow f(cx+d)=g(ax+b)$ for all x

$\Leftrightarrow a(cx+d)+b=c(ax+b)+d$ for all x

$\Leftrightarrow ad+b=cb+d$ [Putting $x=0$ on both sides]

$\Leftrightarrow f(d)=g(b)$

160 **(b)**

Let x be any real number. Then, there exists an integer k such that $k\le x<k+1$

If $k\le x<k+\frac{1}{2}$, then

$\Rightarrow 2k\le 2x<2k+1\Rightarrow[2x]=2k$ and $[x]=k$

$\therefore f(x) = [2x] - 2[x] = 2k - 2k = 0$

If $k + \frac{1}{2} \leq x < k + 1$, then

$2k + 1 \leq 2x < 2k + 2$

$\Rightarrow [2x] = 2k + 1$ and $[x] = k$

$\therefore f(x) = [2x] - 2[x] = 2k + 1 - 2k = 1$

Hence, Range $(f) = \{f(x) : x \in R\} = \{0, 1\}$

161 **(b)**

$f(x)$ is defined, if

$\log_{10}(1 + x^3) > 0 \Rightarrow 1 + x^3 > 10^0 \Rightarrow x^3 > 0 \Rightarrow x > 0 \Rightarrow x \in (0, \infty)$

Hence, domain of $f = (0, \infty)$

162 **(d)**

Since, $(3,3), (6,6), (9,9), (12,12) \in R \Rightarrow R$ is reflexive.

Now, $(6,12) \in R$ but $(12,6) \notin R \Rightarrow R$ is not symmetric.

Also, $(3,6), (6,12) \in R \Rightarrow (3,12) \in R$

$\Rightarrow R$ is transitive.

163 **(a)**

We have,

$f(x+2) - 2f(x+1) + f(x)$

$= a^{x+2} - 2\,a^{x+1} + a^x = a^x(a^2 - 2\,a + 1)$

$= a^x(a-1)^2 = (a-1)^2 f(x)$

So, option (a) holds

It can be easily checked that all other options are not true

164 **(a)**

We have,

$f(x) = \frac{10^x - 10^{-x}}{10^x + 10^{-x}} + 1$

$\therefore fof^{-1}(x) = x$

$\Rightarrow f(f^{-1}(x)) = x$

$\Rightarrow f(y) = x$, where $y = f^{-1}(x)$

$\Rightarrow \frac{10^y - 10^{-y}}{10^y + 10^{-y}} + 1 = x$

$\Rightarrow \frac{10^{2y} - 1}{10^{2y} + 1} + 1 = x$

$\Rightarrow \frac{10^{2y} - 1}{10^{2y} + 1} = x - 1 \Rightarrow \frac{2 \times 10^{2y}}{-2} = \frac{x}{x-2} \Rightarrow 10^{2y} = \frac{x}{2-x}$

$\Rightarrow 2y = \log_{10}\left(\frac{x}{2-x}\right) \Rightarrow y = \frac{1}{2}\log_{10}\left(\frac{x}{2-x}\right)$

$\Rightarrow f^{-1}(x) = \frac{1}{2}\log_{10}\left(\frac{x}{2-x}\right)$

165 **(b)**

We have,

$fog\,(x) = \sqrt{|3^{\tan \pi x} - 3^{1-\tan \tan \pi}| - 2}$

For $fog\,(x)$ to be defined, we must have

$|3^{\tan \pi x} - 3^{1-\tan \pi x}| - 2 \geq 0$

$\Rightarrow \left|3^{\tan \pi x} - \frac{3}{3^{\tan \pi x}}\right| \geq 2$

$\Rightarrow \left|t - \frac{3}{t}\right| \geq 2$, where $t = 3^{\tan \pi x} > 0$

$\Rightarrow t - \frac{3}{t} \geq 2$ or $t - \frac{3}{t} \leq -2$

$\Rightarrow t^2 - 2t - 3 \geq 0$ or $t^2 + 2t - 3 \leq 0$

$\Rightarrow (t-3)(t+1) \geq 0$ or $(t+3)(t-1) \leq 0$

$\Rightarrow t \geq 3$ or $0 < t \leq 1$ $[\because t > 0]$

$\Rightarrow 3^{\tan \pi x} \geq 3$ or, $3^{\tan \pi x} \leq 1$

$\Rightarrow \tan \pi x \geq 1$ or, $\tan \pi x \leq 0$

$\Rightarrow n\pi + \frac{\pi}{4} \leq \pi x < n\pi + \frac{\pi}{2}$ or $n\pi - \frac{\pi}{2} < \pi x < \pi x < n\pi, n \in Z$

$\Rightarrow n\pi + \frac{\pi}{4} \leq \pi x < n\pi + \frac{\pi}{2}$ or, $n\pi + \frac{\pi}{2} \leq \pi x < (n+1)\pi, n \in Z$

$\Rightarrow x \in \left(n + \frac{1}{4}, n + \frac{1}{2}\right) \cup \left(n + \frac{1}{2}, n + 1\right)$

166 **(c)**

Let $f^{-1}(5) = x$. Then,

$f(x) = 5 \Rightarrow 3x - 4 = 5 \Rightarrow x = 3 \Rightarrow f^{-1}(5) = 3$

$\therefore g^{-1}(f^{-1}(5)) = g^{-1}(3)$

Let $g^{-1}(3) = y$. Then, $g(y) = 3 \Rightarrow 3y + 2 = 3 \Rightarrow y = \frac{1}{3}$

$\therefore g^{-1}(f^{-1}(5)) = \frac{1}{3}$

167 **(d)**

We have,

$f(x) + g(x) = e^x$ and $f(x) - g(x) = e^{-x}$

$\Rightarrow f(x) = \frac{e^x + e^{-x}}{2}$ and $g(x) = \frac{e^x - e^{-x}}{2}$

Clearly, $f(-x) = f(x)$ and $g(-x) = -g(x)$ for all $x \in R$

Hence, $f(x)$ is an even function and $g(x)$ is an odd function

168 **(c)**

Given, $f(x) = \frac{1}{2} - \tan\left(\frac{\pi x}{2}\right), -1 < x < 1$

Given, domain of $f(x)$ is $d_1 = (-1, 1)$

For domain of $g(x), 3 + 4x - 4x^2 \geq 0$

$\Rightarrow \quad (2x-3)(2x+1) \leq 0$

$\therefore$ Domain of $g(x)$ is $d_2 = \left[-\frac{1}{2}, \frac{3}{2}\right]$

Hence, domain of $(f+g) = d_1 \cap d_2 = \left[-\frac{1}{2}, 1\right]$

169 **(b)**

Given, $f(x) = 2x^6 + 3x^4 + 4x^2$

Now, $f(-x) = 2(-x)^6 + 3(-x)^4 + 4(-x)^2$

$= 2x^6 + 3x^4 + 4x^2 = f(x)$ $\therefore f(-x) = f(x)$

$\Rightarrow f(x)$ is an even function.

$\Rightarrow f'(x)$ is an odd function.

170 **(b)**

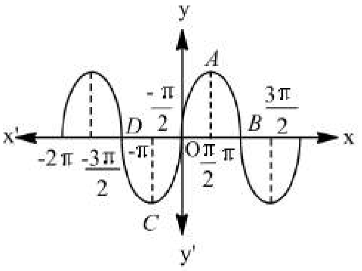
$$f(x)=\sqrt{\cos^{-1}\left(\frac{1-|x|}{2}\right)}$$

$$-1\le\frac{1-|x|}{2}\le 1\Rightarrow -2-1\le -|x|\le 2-1$$

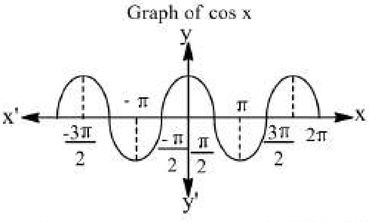
$$\Rightarrow\ -3\le -|x|\le 1\Rightarrow -1\le |x|\le 3\Rightarrow x\in[-3,3]$$

171 **(d)**

Graph of sin x



Graph of cos x



In the given options (a), (b), (c), (e) the curves are decreasing and increasing in the given intervals, so it is not one-to-one function. But in option (d), the curve is only increasing in the given intervals, so it is one-to-one function.

172 **(b)**

We have,

$$f(x)=\log\left(\frac{1+x}{1-x}\right) \text{ and } g(x)=\frac{3x+x^3}{1+3x^2}$$

$$\therefore fog\,(x)=f(g(x))=f\left(\frac{3x+x^3}{1+3x^2}\right)$$

$$\Rightarrow fog\,(x)=\log\left(\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}\right)=\log\frac{(1+x)^3}{(1-x)^3}$$

$$\Rightarrow fog\,(x)=\log\left(\frac{1+x}{1-x}\right)^2$$

$$=3\log\left(\frac{1+x}{1-x}\right)=3f(x)$$

173 **(d)**

For $f(x)$ to be defined $\frac{x-1}{x}\ge 0$

$\Rightarrow x\ge 1$ and $x<0$

$\therefore$ Required interval is $(-\infty,0)\cup[1,\infty)$.

174 **(c)**

If $f(x)=\sin x+\left[\frac{x^2}{a}\right]$ is an odd function, then

$f(-x)=-f(x)$ for all $x\in[-2,2]$

$\Rightarrow -\sin x+\left[\frac{x^2}{a}\right]=-\sin x-\left[\frac{x^2}{a}\right]$ for all $x \in[-2,2]$

$\Rightarrow\left[\frac{x^2}{a}\right]=0$ for all $x\in[-2,2]$

$\Rightarrow 0\le\frac{x^2}{a}<1$ for all $x\in[-2,2]$

$\Rightarrow a>0$ and $a>x^2$ for all $x\in[-2,2]$

$\Rightarrow a>0$ and $a>4\Rightarrow a\in(4,\infty)$

175 **(b)**

(i) aRa, then GCD of a and a is a.

$\therefore R$ is not reflexive.

(ii) $aRb\Rightarrow bRa$

If GCD of a and b is 2, then GCD of b and a is 2.

$\therefore R$ is symmetric.

(iii) $aRa, bRc\nRightarrow cRa$

If GCD of a and b is 2 and GCD of b and c is 2, then it is need not to be GCD of c and a is 2.

$\therefore R$ is not transitive.

176 **(b)**

We have,

$f(x+\lambda)=1+[1+\{1-f(x)\}^5]^{1/5}$

$\Rightarrow f(x+\lambda)-1=[1+\{1-f(x)\}^5]^{1/5}$

$\Rightarrow g(x+\lambda)=[1-\{g(x)\}^5]^{1/5}$, where $g(x)=f(x)-1$

$\Rightarrow g(x+2\lambda)=[1-\{g(x+\lambda)\}^5]^{1/5}$

$\Rightarrow g(x+2\lambda)=[1-[1-\{g(x)\}^5]^{1/5}$

$\Rightarrow g(x+2\lambda)=g(x)$

$\Rightarrow f(x+2\lambda)-1=f(x)-1$ for all $x\in R$

$\Rightarrow f(x+2\lambda)=f(x)$ for all $x\in R$

Hence, $f(x)$ is periodic with period 2λ

177 **(b)**

We observe that $f(x)$ is defined for

$$\log\left(\frac{1}{|\sin x|}\right)\ge 0$$

$$\Rightarrow\frac{1}{|\sin x|}\ge 1 \text{ and } |\sin x|\ne 0$$

$$\Rightarrow|\sin x|\ne 0\quad\left[\because\frac{1}{|\sin x|}\ge 1 \text{ for all } x\right]$$

$\Rightarrow x\ne n\pi, n\in Z$

Hence, domain of $f(x)=R-\{n\pi:n\in Z\}$

178 **(c)**

$$f\left(\frac{x+y}{1+xy}\right)=\log\left(\frac{1+\frac{x+y}{1+xy}}{1-\frac{x+y}{1+xy}}\right)$$

$$=\log\left(\frac{1+xy+x+y}{1+xy-x-y}\right)$$

$$=\log\left(\frac{(1+x)(1+y)}{(1-x)(1-y)}\right)$$

$= \log\left(\frac{1+x}{1-x}\right) + \log\left(\frac{1+y}{1-y}\right)$

$= f(x) + f(y)$

179 **(a)**

It is given that $f(x)$ is defined on $[0, 1]$. Therefore, $f(\tan x)$ exists, if

$0 \leq \tan x \leq 1$

$\Rightarrow n\pi \leq x \leq n\pi + \frac{\pi}{4}, n \in Z \Rightarrow x \in \left[n\pi, n\pi + \frac{\pi}{4}\right], n \in Z$

180 **(d)**

Given, $F(0) = 2, \quad F(1) = 3,$

Since, $F(n+2) = 2F(n) - F(n+1)$

At $n = 0, \quad F(0+2) = 2F(0) - F(1)$

$\Rightarrow \quad F(2) = 2(2) - 3 = 1$

At $n = 1, F(1+2) = 2F(1) - F(2)$

$\Rightarrow \quad F(3) = 2(3) - 1 = 5$

At $n = 2, F(2+2) = 2F(2) - F(3) \Rightarrow F(4) = 2(1) - 5 = -3$

At $n = 3, \quad F(3+2) = 2F(3) - F(4) = 2(5) - (-3)$

$\Rightarrow \quad F(5) = 13$

181 **(b)**

We observe that $\sqrt{\sin^{-1}(\log_2 x)}$ exists for $\sin^{-1}(\log_2 x) \geq 0$ i.e. for $0 \leq \log_2 x \leq 1 \Rightarrow 2^0 \leq x \leq 2 \Rightarrow 1 \leq x \leq 2$

182 **(d)**

We have,

$f(x) = \begin{cases} 1, x \in Q \\ 0, x \notin Q \end{cases}$

We observe that for every rational number T

$f(x+T) = \begin{cases} 1, x \in Q \\ 0, x \notin Q \end{cases}$

But, there is no least position rational number

Hence, $f(x)$ is periodic with indeterminate period

184 **(b)**

We have,

$f(x) = |\cos x| = \sqrt{\frac{1+\cos 2x}{2}}$

Since $\cos x$ is periodic with period 2π. Therefore, $f(x)$ is periodic with period $(2\pi/2) = \pi$

185 **(d)**

We have,

$gof(x) = n\, g(x)$

$\Rightarrow g(f(x)) = n\, g(x) \Rightarrow g(x^n) = n\, g(x) \quad$ ...(i)

Also, $\log x^n = n \log |x| \quad$ ...(ii)

From (i) and (ii), we get $g(x) = \log |x|$

187 **(b)**

Let $y = f(x) = 2^{x(x-1)}$

$\Rightarrow \quad \log_2 y = x^2 - x \Rightarrow x^2 - x - \log_2 y = 0$

$\Rightarrow \quad x = \frac{1 \pm \sqrt{1 + 4\log_2 y}}{2} = \frac{1 + \sqrt{1 + 4\log_2 y}}{2}$

$\left[\because x = \frac{1 - \sqrt{1 + 4(x^2 - x)}}{2} = \frac{1 - (2x-1)}{2} < 0 \text{ domain is not defined}\right]$

188 **(c)**

Given that, $f(x) = |x|$ and $g(x) = [x - 3]$

For $-\frac{8}{3} < x < \frac{8}{5}, 0 \leq f(x) < \frac{8}{5}$

Now, for $0 \leq f(x) < 1$,

$g(f(x)) = [f(x) - 3] = -3$

$[\because -3 \leq f(x) - 3 < -2]$

Again, for $1 \leq f(x) < 16$

$g(f(x)) = -2$

$[\because -2 \leq f(x) - 3 < -14]$

Hence, required set is $\{-3, -2\}$.

189 **(b)**

We have,

$f(x) = \log_{10}\{1 - \log_{10}(x^2 - 5x + 16)\}$

Clearly, $f(x)$ is defined if

$1 - \log_{10}(x^2 - 5x + 16) > 0$ and $x^2 - 5x + 16 > 0$

$\Rightarrow \log_{10}(x^2 - 5x + 16) < 1 \; [\because x^2 - 5x + 16 > 0 \text{ for all } x \in R]$

$\Rightarrow x^2 - 5x + 16 < 10$

$\Rightarrow x^2 - 5x + 6 < 0 \Rightarrow (x-2)(x-3) < 0 \Rightarrow x \in (2, 3)$

190 **(b)**

$f(x) = \sin^4 x + \cos^4 x$

$= (\sin^2 x cos^2 x)^2 - 2\sin^2 x \cos^2 x$

$= 1 - \frac{1}{2}(\sin 2x)^2$

$= \frac{3}{4} + \frac{1}{4}\cos 4x$

$\therefore$ The period of $f(x) = \frac{2\pi}{4} = \frac{\pi}{2}$

191 **(b)**

$\because g(f(x)) = (\sin x + \cos x)^2 - 1$, is invertible (*ie*, bijective)

$\Rightarrow g(f(x)) = \sin 2x$, is bijective

We know $\sin x$ is bijective only when $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Thus, $g(f(x))$ is bijective if, $-\frac{\pi}{2} \leq 2x \leq \frac{\pi}{2}$

$\Rightarrow \quad -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$

192 **(a)**

Here, $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$, to find domain we must have,

$\sin^{-1}(2x) + \frac{\pi}{6} \geq 0$

$\left(\text{but } -\frac{\pi}{2} \le \sin^{-1}\theta \le \frac{\pi}{2}\right)$

$\therefore \quad -\frac{\pi}{6} \le \sin^{-1}(2x) \le \frac{\pi}{2}$

$\Rightarrow \sin\left(-\frac{\pi}{6}\right) \le 2x \le \sin\left(\frac{\pi}{2}\right)$

$\Rightarrow -\frac{1}{2} \le 2x \le 1 \Rightarrow x \in \left[-\frac{1}{4}, \frac{1}{2}\right]$

193 **(d)**

$$f\left(\frac{2x}{1+x^2}\right) = \log\left[\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right] = \log\left(\frac{1+x}{1-x}\right)^2$$

$$\therefore \quad f\left(\frac{2x}{1+x^2}\right) = 2f(x)$$

194 **(c)**

We have, $f(x) = \log_e|\log_e x|$

Clearly, $f(x)$ is defined for all x satisfying

$|\log_e x| > 0 \Rightarrow x \in (0, \infty)$ and $x \ne 1 \Rightarrow x \in (0,1) \cup (1, \infty)$

196 **(c)**

For $f(x)$ to be defined, $\frac{x}{1-|x|} > 0$

ie, $\quad x > 0, \quad 1 - |x| > 0 \quad$ or $x < 0, \quad 1 - |x| < 0$

$\Rightarrow \quad x \in (0,1) \quad$ or $x \in (-\infty, -1)$

$\therefore \quad x \in (-\infty, -1) \cup (0,1)$

197 **(a)**

Given,

$$f(x) = \begin{cases} [x] & \text{if } -3 < x \le -1 \\ |x| & \text{if } -1 < x < 1 \\ |[x]| & \text{if } 1 \le x < 3 \end{cases}$$

When $-3 < x \le -1, \quad f(x) = [x] \quad \Rightarrow \quad f(x) < 0$

When $-1 < x < 1, \quad f(x) = |x| \quad \Rightarrow \quad f(x) > 0$

When $1 \le x < 3, \; f(x) = |[x]| \Rightarrow \quad f(x) > 0$

$\therefore$ The set $\;(x : f(x) \ge 0) = (-1, 3)$.

198 **(a)**

$$(fof)x = f\left(\frac{x}{x-1}\right)$$

$$= \frac{\frac{x}{x-1}}{\left(\frac{x}{x-1}\right) - 1} = x$$

$$\Rightarrow \quad (fofof)x = f(fof)x = f(x) = \frac{x}{x-1}$$

$$\therefore \quad (fofof \dots 19 \text{ times})(x) = \frac{x}{x-1}$$

199 **(a)**

For the given function to be defined, we must have

$x - 4 \ge 0$ and $6 - x \ge 0$

$\Rightarrow x \ge 4$ and $x \le 6 \Rightarrow x \in [4, 6]$

$\therefore$ The domain of $f(x)$ is $[4, 6]$

200 **(c)**

We have,

$f(n) =$ Sum of positive divisors of n

$\therefore f(2^k \times 3) =$ Sum of positive divisors of $2^k \times 3$

$$\Rightarrow f(2^k \times 3) = \sum_{r=0}^{k} (2^r \times 3)$$

$$\Rightarrow f(2^k \times 3) = 3 + 2 \times 3 + 2^2 \times 3 + \dots + 2^k \times 3$$

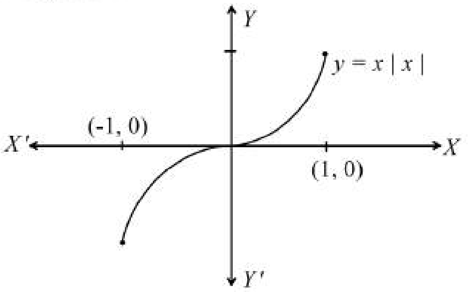
$$\Rightarrow f(2^k \times 3) = 3\left(\frac{2^{k+1} - 1}{2 - 1}\right) = 3(2^{k+1} - 1)$$

201 **(a)**

We have,

$$f(x) = x|x| = \begin{cases} x^2, & 0 \le x \le 1 \\ -x^2, & -1 \le x < 0 \end{cases}$$

The graph of $f(x)$ is as shown below. Clearly, it is a bijection



202 **(b)**

Foe domain of given function

$$-1 \le \log_2 \frac{x^2}{2} \le 1$$

$\Rightarrow \quad 2^{-1} \le \frac{x^2}{2} \le 2 \Rightarrow 1 \le x^2 \le 4$

$\Rightarrow \quad |x| \le 2$ and $|x| \ge 1$

$\Rightarrow \quad x \in [-2, 2] - (-1, 1)$

203 **(c)**

Given, $\quad f(x) = ax + b, \quad g(x) = cx + d$

$\because \quad f\{g(x)\} = g\{f(x)\}$

$\Rightarrow \quad f(cx + d) = g(ax + b)$

$\Rightarrow \quad a(cx + d) + b = c(ax + b) + d$

$\Rightarrow \quad ad + b = bc + d$

$\Rightarrow \quad f(d) = g(b)$

204 **(c)**

Since $\phi(x) = \sin^4 x + \cos^4 x$ is periodic with period $\pi/2$

$\therefore f(x) = \sin^4 3x + \cos^4 3x$ is periodic with period $\frac{1}{3}\left(\frac{\pi}{2}\right) = \frac{\pi}{6}$

205 **(b)**

We have,

$$f(x) = \log\left(\frac{1+x}{1-x}\right) \text{ and } g(x) = \frac{3x + x^3}{1 + 3x^2}$$

$\therefore fog\,(x) = f(g(x))$

$$\Rightarrow fog\,(x) = f\left(\frac{3\,x + x^3}{1 + 3\,x^2}\right)$$

$$= \log\left(\frac{1 + \frac{3\,x+x^3}{1+3\,x^2}}{1 - \frac{3\,x+x^3}{1+3\,x^2}}\right)$$

$$= \log\left\{\frac{(1 + x)^3}{(1 - x)^3}\right\}$$

$$\Rightarrow fog\,(x) = \log\left(\frac{1 + x}{1 - x}\right)^3 = 3\log\left(\frac{1 + x}{1 - x}\right)$$

$$= 3f(x)$$

206 **(b)**

For choice (a), we have

$f(x) = f(y); x, y \in [-1, \infty)$

$\Rightarrow |x + 1| = |y + 1| \Rightarrow x + 1 = y + 1 \Rightarrow x = y$

So, f is an injection

For choice (b), we obtain

$g(2) = \frac{5}{2}$ and $g\left(\frac{1}{2}\right) = \frac{5}{2}$

So, $g(x)$ is not injective

It can be easily seen that the functions in choices in options (c) and (d) are injective maps

207 **(b)**

Given, $f(x) = x - [x], g(x) = [x]$ for $x \in R$.

$\therefore \quad f(g(x)) = f([x])$

$= [x] - [x]$

$= 0$

208 **(a)**

We have,

$$f(x) = \sqrt{\frac{\log_{0.3}|x - 2|}{|x|}}$$

We observe that $f(x)$ assumes real values, if

$\frac{\log_{0.3}|x - 2|}{|x|} \geq 0$ and $|x - 2| > 0$

$\Rightarrow \log_{0.3}|x - 2| \geq 0$ and $x \neq 0, 2$

$\Rightarrow |x - 2| \leq 1$ and $x \neq 0, 2$

$\Rightarrow x \in [1, 3]$ and $x \neq 2 \Rightarrow x \in [1, 2) \cup (2, 3]$

209 **(d)**

Since $g(x) = 3\sin x$ is a many-one function.

Therefore, $f(x) - 3\sin x$ is many-one

Also, $-1 \leq \sin x \leq 1$

$\Rightarrow -3 \leq -3\sin x + 3$

$\Rightarrow 2 \leq 5 - 3\sin x \leq 8$

$\Rightarrow 2 \leq f(x) \leq 8 \Rightarrow$ Range of $f(x) = [2, 8] \neq R$

So, $f(x)$ is not onto

Hence, $f(x)$ is neither one-one nor onto

210 **(a)**

We have,

$f(x + 2y, x - 2y) = xy \quad$ ....(i)

Let $x + 2y = u$ and $x - 2y = v$. Then,

$x = \frac{u + v}{2}$ and $y = \frac{u - v}{4}$

Substituting the values of x and y in (i), we obtain

$f(u, v) = \frac{u^2 - v^2}{2}$ and $f(x, y) = \frac{x^2 - y^2}{8}$

211 **(c)**

Given, $f(x) = y = (1 - x)^{1/3}$

$\Rightarrow \quad y^3 = 1 - x$

$\Rightarrow \quad x = 1 - y^3$

$\therefore \quad f^{-1}(x) = 1 - x^3$

212 **(a)**

We have, $f(x + 2y, x - 2y) = xy$

...(i)

Let $\quad x + 2y = u$ and $x - 2y = v$

Then, $\quad x = \frac{u+v}{2}$ and $y = \frac{u-v}{4}$

Subtracting the values of x and y in Eq. (i), we obtain

$f(u, v) = \frac{u^2 - v^2}{8} \Rightarrow f(x, y) = \frac{x^2 - y^2}{8}$

213 **(d)**

Given, $f(x) = 5^{x(x-4)}$ for $f: [4, \infty[\to [4, \infty[$

At $x = 4$

$f(x) = 5^{4(4-4)} = 1$

Which is not lie in the interval $[4, \infty[$

$\therefore$ Function is not bijective.

Hence, $f^{-1}(x)$ is not defined.

214 **(b)**

Given, $f(x) = x^3 + 3x - 2$

On differentiating w.r.t. x, we get

$f'(x) = 3x^2 + 3$

Put $f'(x) = 0 \Rightarrow 3x^2 + 3 = 0$

$\Rightarrow \quad x^2 = -1$

$\therefore$ $f(x)$ is either increasing or decreasing.

At $x = 2, f(2) = 2^3 + 3(2) - 2 = 12$

At $x = 3, f(3) = 3^3 + 3(3) - 2 = 34$

$\therefore f(x) \in [12, 34]$

215 **(b)**

We have,

$f(\theta) = \sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$\therefore f(\theta)$ is periodic with period $\frac{2\pi}{2} = \pi$

216 **(c)**

Since, period of $\cos nx = \frac{2\pi}{n}$

And period of $\sin\left(\frac{x}{n}\right) = 2n\pi$

$\therefore$ Period of $\frac{\cos nx}{\sin\left(\frac{x}{n}\right)}$ is $2n\pi$

$\Rightarrow \quad 2n\pi = 4\pi \quad \Rightarrow \quad n = 2$

217 **(c)**

Given, $f(x) = x^3 + 5x + 1$

Now, $f'(x) = 3x^2 + 5 > 0, \forall x \in R$

$\therefore f(x)$ is strictly increasing function.

$\therefore f(x)$ is one-one function.

Clearly, $f(x)$ is a continous function and also increasing on R,

$\lim_{x\to-\infty} f(x) = -\infty$ *and* $\lim_{x\to\infty} = \infty$

$\therefore f(x)$ takes every value between $-\infty$ and ∞

Thus, $f(x)$is onto function.

218 **(c)**

The function $f(x) = \frac{1}{2-\cos 3x}$ is defined for all $x \in R$. Therefore, domain of $f(x)$ is R

Let $f(x) = y$. Then,

$\frac{1}{2 - \cos 3x} = y$ and $y > 0$

$\Rightarrow 2 - \cos 3x = \frac{1}{y}$

$\Rightarrow \cos 3x = \frac{2y - 1}{y} \Rightarrow x = \frac{1}{3}\cos^{-1}\left(\frac{2y-1}{y}\right)$

Now,

$x \in R$, if

$-1 \le \frac{2y-1}{y} \le 1$

$\Rightarrow -1 \le 2 - \frac{1}{y} \le 1$

$\Rightarrow -3 \le -\frac{1}{y} \le -1$

$\Rightarrow 3 \ge \frac{1}{y} \ge 1 \Rightarrow \frac{1}{3} \le y \le 1 \Rightarrow y \in [1/3, 1]$

219 **(c)**

Given, $A = \{2, 3, 4, 5, \dots, 16, 17, 18\}$

And $(a, b) = (c, d)$

$\therefore$ Equivalence class of $(3, 2)$ is

$\{(a, b) \in A \times A: (a, b)R\ (3, 2)\}$

$= \{(a, b) \in A \times A: 2a = 3b\}$

$= \left\{(a, b) \in A \times A: b = \frac{2}{3}a\right\}$

$\left\{\left(a, \frac{2}{3}a\right): a \in A \times A\right\}$

$= \{(3, 2), (6, 4), (9, 6), (12, 8), (15, 10), (18, 12)\}$

$\therefore$ Number of ordered pairs of the equivalence class=6.

220 **(c)**

Given function is $f(n) = 8 -^n P_{n-4}, 4 \le n \le 6$. It is defined, if

1. $8 - n > 0 \Rightarrow n < 8$ ... (i)
2. $n - 4 \ge 0 \Rightarrow n \ge 4$ ... (ii)
3. $n - 4 \le 8 - n \Rightarrow n \le 6$ ... (iii)

From Eqs. (i), (ii) and (iii), we get $n = 4, 5, 6$

Hence, range of $f(n) = \{^4P_0, ^3P_1, ^2P_2\} = \{1, 3, 2\}$

221 **(c)**

Clearly, $X = R^+$ and $Y = R$

222 **(b)**

Given, $f(x).f\left(\frac{1}{2}\right) = f(x) + f\left(\frac{1}{x}\right)$

Let $f(x) = x^n \pm 1$, where $n \in I$.

Now, $f(4) = 65$

Case I

Let $f(x) = x^n + 1$

$\Rightarrow f(4) = 4^n + 1$

$\Rightarrow 65 = 4^n + 1$

$\Rightarrow n = 3$

Case II

Let $f(x) = x^n - 1$

$\Rightarrow f(4) = 4^n - 1 \Rightarrow 65 = 4^n - 1$

$\Rightarrow 4^n = 66$

The quality does not hold true for $n \in Z$.

Therefore, $f(x) = x^3 + 1$

Now, $f(6) = 6^3 + 1 = 216 + 1 = 217$

223 **(b)**

Since, the graph is symmetrical about the line= $x = 2$

$\Rightarrow f(2 + x) = f(2 - x)$

224 **(c)**

We have,

$f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$ and $g(x) = x(1 - x^2)$

$\therefore fog\,(x) = f(g(x))$

$\Rightarrow fog\,(x) = \begin{cases} -1, & \text{if } g(x) < 0 \\ 0, & \text{if } g(x) = 0 \\ 1, & \text{if } g(x) > 0 \end{cases}$

$\Rightarrow fog\,(x) = \begin{cases} -1, & \text{if } x \in (-1, 0) \cup (1, \infty) \\ 0, & \text{if } x = 0, \pm 1 \\ 1, & \text{if } x \in (-\infty, -1) \cup (0, 1) \end{cases}$

225 **(b)**

Reflexive xRx

Since, $x^2 = x.x$

$x^2 = xy$

Transitive, $xRy \Rightarrow x^2 = xy$

And $yRz \Rightarrow y^2 = yz$

Now, $x^2y^2 = xy^2z \Rightarrow x^2 = xz$

$\Rightarrow xRz$

$\therefore$ It is transitive.

226 **(c)**

We have,

$f(x) = \sin\left(\frac{\pi x}{n-1}\right) + \cos\left(\frac{\pi x}{n}\right), n \in Z, n > 2$

Since $\sin\left(\frac{\pi x}{n-1}\right)$ and $\cos\left(\frac{\pi x}{n}\right)$ are periodic functions with period $2(n - 1)$ and $2n$ respectively.

Therefore, $f(x)$ is periodic with period equal to LCM of $(2n, 2(n - 1)) = 2n(n - 1)$

227 **(b)**

Let $g(x)$ be the even extension of $f(x)$ on $[-4,4]$

Then,

$$g(x)=\begin{cases} f(x) \text{ for } x\in[-4,0] \\ f(-x) \text{ for } x\in[0,4]\end{cases}$$

$$\Rightarrow g(x)=\begin{cases} e^{x}+\sin x \text{ for } x\in[-4,0] \\ e^{-x}+\sin(-x) \text{ for } x\in[0,4]\end{cases}$$

$$\Rightarrow g(x)=\begin{cases} e^{x}+\sin x \text{ for } x\in[-4,0] \\ e^{-x}-\sin x \text{ for } x\in[0,4]\end{cases}$$

$\Rightarrow g(x)=e^{-|x|}-\sin|x|$ for $x\in[-4,4]$

228 **(d)**

Clearly, $f(x)$ is an even function and $f(x)<0$ for all $x>0$

Therefore, the graph of $f(x)$ lies in the third and fourth quadrants

229 **(d)**

The given function is

$$f(x)=\sqrt{1-2x}+2\sin^{-1}\left(\frac{3x-1}{2}\right)$$

For domain of $f(x)$, $1-2x\geq 0$ and $-1\leq\frac{3x-1}{2}\leq 1$

$\Rightarrow \quad x\leq\frac{1}{2}$ and $-2\leq 3x-1\leq 2$

$\Rightarrow \quad x\leq\frac{1}{2}$ and $-\frac{1}{3}\leq x\leq 1$

$\therefore$ Domain of $f(x)=\left[-\frac{1}{3},\frac{1}{2}\right]$

230 **(c)**

We have,

$f(x)=\log_{(x+3)}(x^2-1)$

Clearly, $f(x)$ is defined for x satisfying the following conditions

(i) $x^2-1>0$ (ii) $x+3>0$ and $x+3\neq 1$

Now, $x^2-1>0\Rightarrow x\in(-\infty,-1)\cup(1,\infty)$

and,

$x+3>0$ and $x+3\neq 1\Rightarrow x>-3$ and $x=-2$

$\Rightarrow x\in(-3,-2)\cup(-2,\infty)$

Hence, the domain of $f(x)$ is $(-3,-2)\cup(-2,-1)\cup(1,\infty)$

231 **(b)**

$x^2-6x+7=(x-3)^2-2$

Obviously, minimum value is -2 and maximum is ∞.

232 **(d)**

We have,

$fof^{-1}(x)=x$

$\Rightarrow f\left(f^{-1}(x)\right)=x$

$\Rightarrow f(y)=x$ where $y=f^{-1}(x)$

$$\Rightarrow\frac{e^y-e^{-y}}{e^y+e^{-y}}+2=x\Rightarrow\frac{e^y-e^{-y}}{e^y+e^{-y}}=x-2$$

$$\Rightarrow\frac{2e^y}{-2e^{-y}}=\frac{x-1}{x-3}$$

$$\Rightarrow e^{2y}=\frac{x-1}{3-x}$$

$$\Rightarrow y=\frac{1}{2}\log\left(\frac{x-1}{3-x}\right)$$

$$\Rightarrow f^{-1}(x)=\frac{1}{2}\log\left(\frac{x-1}{3-x}\right)$$

233 **(b)**

$$f(x)=\frac{4^x}{4^x+2}$$

$$\therefore\quad f(1-x)+f(x)=\frac{4^{1-x}}{4^{1-x}+2}+\frac{4^x}{4^x+2}$$

$$=\frac{4}{4+2.4^x}+\frac{4^x}{4^x+2}=\frac{2}{2+4^x}+\frac{4^x}{4^x+2}=1$$

By putting $x=\frac{1}{97},\frac{2}{97},\frac{3}{97},\dots.\frac{48}{97}$

And adding, we get

$$f\left(\frac{1}{97}\right)+f\left(\frac{2}{97}\right)+\dots+f\left(\frac{96}{97}\right)=48$$

234 **(c)**

Given, $f(x)=\frac{2\sin 8x\cos x-2\sin 6x\cos 3x}{2\cos 2x\cos x-2\sin 3x\sin 4x}$

$$=\frac{(\sin 9x+\sin 7x)+(\sin 9x+\sin 3x)}{(\cos 3x+\cos x)+(\cos 7x-\cos x)}$$

$$=\frac{\sin 7x-\sin 3x}{\cos 7x+\cos 3x}$$

$$=\frac{2\cos 5x\sin 2x}{2\cos 2x\cos 5x}=\tan 2x$$

$\therefore$ Period of $f(x)=\frac{\pi}{2}$

235 **(d)**

$gof=g\{f(x)\}=g(x^2)=x^2+5$

236 **(b)**

We have,

$f(x)=\log_{2x-5}(x^2-3x-10)$

For $f(x)$ to be defined, we must have

$x^2-3x-10>0$, $2x-5>0$ and $2x-5\neq 1$

$\Rightarrow (x-5)(x+2)>0, x>\frac{5}{2}$ and $\frac{5}{2}$ and $x\neq 3$

$\Rightarrow x>5\Rightarrow x\in(5,\infty)$

237 **(c)**

Since, $f(x)$ is an even function therefore its values is always greater than equal to 0 and we know

$x^2<x^2+1$ or $\frac{x^2}{x^2+1}<1$

$\therefore$ Required range is $[0,1)$.

238 **(d)**

We have,

$f(x^2)=|x^2-1|\neq|x-1|^2=[f(x)]^2$

$f(|x|)=\big||x|-1\big|\neq|x-1|=|f(x)|$

And,

$f(x+y)=|x+y-1|\neq|x-1|+|y-1|$

$=f(x)+f(y)$

Hence, none of the above given option is true

239 **(d)**

We have,
$f(x+2)-2f(x+1)+f(x)$
$=a^{x+2}-2\,a^{x+1}+a^x$
$=a^x(a^2-2\,a+1)=a^x(a-1)^2=(a-1)^2 f(x)$
So, option (a) holds
It can be easily checked that options (b) and (c) are also true but option (d) is not true

240 **(b)**
It can be easily seen that $f: A \to A$ is a bijection.
Let $f(x)=y$. Then,
$f(x)=y$
$\Rightarrow x(2-x)=y$
$\Rightarrow x^2-2x+y=0$
$\Rightarrow x^2-2x+y=0$
$\Rightarrow x=\dfrac{2\pm\sqrt{4-4y}}{2}$
$\Rightarrow x=1\pm\sqrt{1-y}$
$\Rightarrow x=1-\sqrt{1-y}$ $\quad [\because x\le 1]$
$\Rightarrow f^{-1}(y)=1-\sqrt{1-y}$
Hence, $f^{-1}: A\to A$ is defined as $f^{-1}(x)=1-\sqrt{1-x}$

241 **(d)**
We observe that
Period of $\sin\frac{\pi x}{2}$ is $\frac{2\pi}{\pi/2}=4$, Period of $\cos\frac{\pi x}{3}$ is $\frac{2\pi}{\pi/3}=6$,
and,
Period of $\tan\dfrac{\pi x}{4}$ is $\dfrac{\pi}{\pi/4}=4$
$\therefore$ Period of $f(x)=$ LCM of $(4,6,4)=12$

242 **(c)**
We have,
$f(x)=\lim\limits_{x\to\infty}\dfrac{x^n+x^{-n}}{x^n+x^{-n}}$
$\Rightarrow f(x)=\lim\limits_{x\to\infty}\dfrac{x^{2n}-1}{x^{2n}+1}=\dfrac{0-1}{0+1}=-1$, if $-1<x<1$
If $|x|>1$, then $x^{2n}\to\infty$ as $n\to\infty$
$\therefore f(x)=\lim\limits_{x\to\infty}\dfrac{1-\frac{1}{x^{2n}}}{1+\frac{1}{x^{2n}}}=\dfrac{1-0}{1+1},=1$, if $|x|>1$
If $|x|=1$, then $x^{2n}=1$
$\therefore f(x)=\lim\limits_{x\to\infty}\dfrac{x^{2n}-1}{x^{2n}+1}=\dfrac{1-1}{1+1}=0$
Thus, we have
$$f(x)=\begin{cases}-1, & \text{if } |x|<1\\ 0, & \text{if } |x|=1\\ 1, & \text{if } |x|>1\end{cases}$$

243 **(c)**
$R=\{(1,3),(4,2),(2,4),(2,3),(3,1)\}$ is a relation on
$A=\{1,2,3,4\}$, then
(a) since, $(2,4)\in R$ and $(2,3)\in R$, so R is not a function.
(b) since, $(1,3)\in R$ and $(3,1)\in R$ but $(1,1)\notin R$. So, R is not transitive.
(c) since, $(2,3)\in R$ but $(3,2)\notin R$, so R is not symmetric.
(d) since, $(4,4)\notin R$, so R is not reflexive.

244 **(a)**
We have,
$f(x)={}^{16-x}C_{2x-1}+{}^{20-3x}P_{4x-5}$
Clearly, $f(x)$ is defined, if
$16-x\ge 2x-1>0, 20-3x\ge 4x-5>0$ and $x\in Z$
$\Rightarrow x\in\{1,2,3,4,5\}, x\in\{2,3\}$ and $x\in Z$
$\Rightarrow x\in\{2,3\}$
$\therefore$ Domain $(f)=\{2,3\}$

245 **(d)**
Given, $f(x)=e^{2ix}$ and $f: R\to C$. Function $f(x)$ is not one-one, because after some values of $x(ie,\pi)$ it will give the same values.
Also, $f(x)$ is not onto, because it has minimum and maximum values $-1-i$ and $1+i$ respectively.

246 **(a)**
For $f(x)$ to be defined,
$x-4\ge 0$ and $6-x\ge 0 \quad\Rightarrow\quad x\ge 4$ and $x\le 6$
Therefore, the domain is [4, 6].

247 **(d)**
We have,
$hogof\ (x)=\cos^{-1}(|\sin x|)$
and, $fogoh\ (x)=\sin^2\left(\sqrt{\cos^{-1}x}\right)$
Clearly, $hogof\ (x)\ne fogoh(c)$
Thus, option (a) is not correct
Now,
$gofoh\ (x)=|\sin(\cos^{-1}x)|$
$=\left|\sin\left(\sin^{-1}\sqrt{1-x^2}\right)\right|=\sqrt{1-x^2}$
and, $fohog\ (x)=\sin^2(\cos^{-1}\sqrt{x})$
$=1-\cos^2(\cos^{-1}\sqrt{x})$
$\Rightarrow fohog\ (x)=1-\left\{\cos\left(\cos^{-1}\sqrt{x}\right)\right\}^2=1-x$
$\therefore gofoh\ (x)\ne fohog(x)$
Thus, option (b) is correct
Also,
$hogof(x)=\cos^{-1}(|\sin x|)$ and, $fohog\ (x)=1-x$
$\therefore hogof(x)\ne fohog(x)$

Thus, option (c) is not correct

Hence, option (d) is correct

248 **(a)**

We have,

$$f(x)=\frac{2^x+2^{-x}}{2}$$

$$\therefore f(x+y)f(x-y)$$

$$=\frac{2^{x+y}+2^{-x-y}}{2}\times\frac{2^{x-y}+2^{-x+y}}{2}$$

$$\Rightarrow f(x+y)f(x-y)=\frac{2^{2x}+2^{-2y}+2^{2y}+2^{-2x}}{4}$$

$$\Rightarrow f(x+y)f(x-y)$$

$$=\frac{1}{2}\left(\frac{2^{2x}+2^{-2x}}{2}+\frac{2^{2y}+2^{-2y}}{2}\right)$$

$$\Rightarrow f(x+y)\,f\,(x-y)=\frac{1}{2}\{f(2x)+f\,(2y)\}$$

249 **(b)**

$$R=\{(a,b): a,b\in N, a-b=3\}$$
$$=\{[(n+3),n]: n\in N\}$$
$$=\{(4,1),(5,2),(6,3),\dots\}$$

250 **(a)**

Clearly, $f(x)=\sin^{-1}\left\{\log_3\left(\frac{x}{3}\right)\right\}$ exists if

$$-1\le\log_3\left(\frac{x}{3}\right)\le 1\Leftrightarrow 3^{-1}\le\frac{x}{3}\le 3^1\Leftrightarrow 1\le x\le 9$$

Hence, domain of $f(x)$ is $[1,9]$

251 **(c)**

For $f(x)$ to be defined, we must have

$$\frac{\sqrt{4-x^2}}{1-x}>0, 4-x^2>0 \text{ and } 1-x\ne 0$$

$$\Rightarrow 1-x>0, 4-x^2>0 \text{ and } 1-x\ne 0$$

$$\Rightarrow x<1, x\in(-2,2) \text{ and } x\ne 1\Rightarrow x\in(-2,1)$$

$\therefore$ Domain $(f)=(-2,1)$

Now, for $x\in(-2,1)$, we have

$$-\infty<\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)<\infty$$

$$\Rightarrow -1\le\sin\left\{\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right\}\le 1\Rightarrow -1\le f(x)\le 1$$

Hence, Range $(f)=[-1,1]$

252 **(a)**

Given, $f(x)=\frac{ax+b}{cx+d}$ and $fof(x)=x$

$$\Rightarrow f\left(\frac{ax+b}{cx+d}\right)=x$$

$$\Rightarrow \frac{a\left(\frac{ax+b}{cx+d}\right)+b}{c\left(\frac{ax+b}{cx+d}\right)+d}=x$$

$$\Rightarrow \frac{x(a^2+bc)+ab+bd}{x(ac+cd)+bc+d^2}=x$$

$$\Rightarrow d=-a$$

253 **(c)**

If $f: C\to C$ given by $f(x)=\frac{ax+b}{cx+d}$ is a constant function, then

$f(x)=$ Constant $(=\lambda$, say$)$ for all $x\in C$

$$\Rightarrow\frac{ax+b}{cx+d}=\lambda \text{ for all } x\in C$$

$$\Rightarrow (a-\lambda c)\,x+(b-\lambda d)=0 \text{ for all } x\in C$$

$$\Rightarrow a-\lambda c=0 \text{ and } b-\lambda d=0\Rightarrow\frac{a}{c}=\frac{b}{d}\Rightarrow ad=bc$$

254 **(d)**

Periods of $\sin\lambda\,x+\cos\lambda\,x$ and $|\sin x|+|\cos x|$ are $\frac{2\pi}{\lambda}$ and $\frac{\pi}{2}$ respectively

$$\therefore\frac{\pi}{2}=\frac{2\pi}{\lambda}\Rightarrow\lambda=4$$

255 **(b)**

We have, $f(x)=\sqrt{\log_{16}x^2}$

Clearly, $f(x)$ exists, if

$$\log_{16}x^2\ge 0\Rightarrow x^2\ge 1\Leftrightarrow |x|\ge 1$$

256 **(b)**

Since, $f(x)$ is an even function, therefore $f'(x)$ is an odd function

ie, $\quad f'(-e)=-f'(e)$

$\therefore\quad f'(e)+f'(-e)=0$

257 **(c)**

We have,

$$f(x)=\log\left(\frac{1+x}{1-x}\right)$$

$$\therefore f\left(\frac{2x}{1+x^2}\right)=\log\left\{\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right\}=\log\left(\frac{x+1}{1-x}\right)^2$$

$$\Rightarrow f\left(\frac{2x}{1+x^2}\right)=\log\left(\frac{1+x}{1-x}\right)=2\,f\,(x)$$

258 **(c)**

$$f(x)=\cos^2x+\sin^4x=1-\cos^2x+\cos^4x$$

$$\Rightarrow f(x)=\left(\cos^2x-\frac{1}{2}\right)^2+\frac{3}{4}\ge\frac{3}{4}\quad\text{for all } x$$

Also, $f(x)=\cos^2x+\sin^4x\le\cos^2x+\sin^2x=1$

$\therefore$ Range $(f)=[3/4,1]$

Hence, $f(R)=[3/4,1]$

259 **(d)**

For domain of given function

$$-1\le\log_2\left\{\frac{x}{12}\right\}\le 1$$

$$\Rightarrow 2^{-1}\le\frac{x}{12}\le 2$$

$$\Rightarrow 6\le x\le 24$$

$$\Rightarrow x\in[6,24]$$

260 **(d)**

Given, $f(x)=4^{-x^2}+\cos^{-1}\left(\frac{x}{2}-1\right)+\log(\cos x)$

Here, 4^{-x^2} is defined for $\left\{-\frac{\pi}{2},\frac{\pi}{2}\right\}$, $\cos^{-1}\left(\frac{x}{2}-1\right)$ is defined,

If $-1 \le \frac{x}{2}-1 \le 1 \Rightarrow 0 \le x \le 4$

And $\log(\cos x)$ is defined, if $\cos x > 0$

$\Rightarrow -\frac{\pi}{2} < x < \frac{\pi}{2}$

Hence, $f(x)$ is defined for $x \in \left[0,\frac{\pi}{2}\right]$

261 **(a)**

Let $f^{-1}(x) = y$. Then,

$x = f(y) \Rightarrow x = 3y - 4 \Rightarrow y = \frac{x+4}{3}$

$\therefore f^{-1}(x) = y \Rightarrow f^{-1}(x) = \frac{x+4}{3}$

262 **(d)**

Here, we have to find the range of the function which is $[-1/3, 1]$

263 **(a)**

For $f(x)$ to be real, we must have

$x > 0$ and $\log_{10} x \neq 0$

$\Rightarrow x > 0$ and $x \neq 1 \Rightarrow x > 0$ and $x \neq 1 \Rightarrow x \in (0,1) \cup (1,\infty)$

264 **(a)**

Let $W = \{cat, toy, you, \dots\}$

Clearly, R is reflexive and symmetric but not transitive.

[Since, ${}_{cat}R_{toy}, {}_{toy}R_{you} \nRightarrow {}_{cat}R_{you}$]

265 **(c)**

Given, $f(x) = \frac{ax+b}{cx+d}$

It reduces the constant function if

$$\frac{a}{c} = \frac{b}{d} \Rightarrow \quad ad = bc$$

267 **(c)**

Since, the relation R is defined as

$R = \{(x,y)|x,y$ are real numbers and $x = wy$ for some rational number $w\}$

(i) **Reflexive** $xRx \Rightarrow x = wx$

$\therefore \quad w = 1 \in$ Rational number

$\Rightarrow$ The relation R is reflexive.

(ii) **Symmetric** $xRy \Rightarrow yRx$

As $0R1$

$\Rightarrow \quad 0 = 0\,(1)$ but $1R0 \Rightarrow 1 = w.(0)$,

Which is not true for any rational number

$\Rightarrow$ The relation R is not symmetric

Thus, R is not equivalent relation.

Now, for the relation S is defined as

$S = \left\{\left(\frac{m}{n},\frac{m}{n}\right)\right|$

m, n, p and $q \in$ integers such that $n, q \neq 0$ and $qm = pn\}$

(i) **Reflexive** $\frac{m}{n} S \frac{m}{n} \Rightarrow mn = mn$ (True)

$\Rightarrow$ The relation S is reflexive

(ii) **Symmetric** $\frac{m}{n} S \frac{p}{q} \Rightarrow mq = np$

$\Rightarrow np = mq \Rightarrow \frac{p}{q} S \frac{m}{n}$

$\Rightarrow$ The relation S is symmetric.

(iii) **Transitive** $\frac{m}{n} S \frac{p}{q}$ and $\frac{p}{q} S \frac{r}{s}$

$\Rightarrow mq = np$ and $ps = rq$

$\Rightarrow mq.ps = np.rq$

$\Rightarrow ms = nr \quad \Rightarrow \frac{m}{n} = \frac{r}{s} \Rightarrow \frac{m}{n} S \frac{r}{s}$

$\Rightarrow$ The relation S is transitive

$\Rightarrow$ The relation S is equivalent relation.

268 **(a)**

We know that $\tan x$ has period π. Therefore, $|\tan x|$ has period $\frac{\pi}{2}$. Also, $\cos 2x$ has period π.

Therefore, period of $|\tan x| + \cos 2x$ is π.

Clearly, $2\sin\frac{\pi x}{3} + 3\cos\frac{2\pi x}{3}$ has its period equal to the LCM of 6 and 3 i.e., 6

$6\cos(2\pi x + \pi/4) + 5\sin(\pi x + 3\pi/4)$ has period 2

The function $|\tan 4x| + |\sin 4x|$ has period $\frac{\pi}{2}$

269 **(a)**

Let $y = f(x) = \sqrt{(x-1)(3-x)}$

$\Rightarrow \quad x^2 - 4x + 3 + y^2 = 0$

This is a quadratic in x, we get

$$x = \frac{+4 \pm \sqrt{16 - 4(3+y^2)}}{2(1)} = \frac{4 \pm 2\sqrt{1-y^2}}{2(1)}$$

Since, x is real, then $1 - y^2 \ge 0 \Rightarrow -1 \le y \le 1$

But $f(x)$ attains only non-negative values.

Hence, $y = f(x) = [0, 1]$

270 **(d)**

$\{(z,b),(y,b),(a,d)\}$ is not a relation from A to B because $a \notin A$

272 **(a)**

For $x \ge 1$, we have

$x \le x^2 \Rightarrow \min\{x,x^2\} = x$

For $0 \le x < 1$, we have,

$x^2 < x \Rightarrow \min\{x,x^2\} = x^2$

For $x < 0$, we have

$x < x^2 \Rightarrow \min\{x,x^2\} = x$

Hence, $f(x) = \min\{x,x^2\} = \begin{cases} x, & x > 1 \\ x^2, & 0 \le x < 1 \\ x, & x < 0 \end{cases}$

ALITER Draw the graphs of $y = x$ and $y = x^2$ to obtain $f(x)$

273 **(a)**

Clearly, mapping f given in option (a) satisfies the given conditions

274 **(b)**

Given, $f(x) = e^{\sqrt{5x-3-2x^2}}$

For domain of $f(x)$

$$2x^2 - 5x + 3 \le 0$$

$$\Rightarrow (2x-3)(x-1) \le 0$$

$$\Rightarrow 1 \le x \le \frac{3}{2}$$

$\therefore$ Domain of $f(x) = \left[1, \frac{3}{2}\right]$.

275 **(d)**

Given, $f(x) = x + \sqrt{x^2}$

Since, this function is not defined

276 **(a)**

We have,

$$f(x) = \frac{\sin^4 x + \cos^2 x}{\sin^2 x + \cos^4 x}$$

$$\Rightarrow f(x) = \frac{(1-\cos^2 x)^2 + \cos^2 x}{1 - \cos^2 x + \cos^4 x} = 1 \quad \text{for all } x \in R$$

$\therefore f(2010) = 1$

277 **(c)**

We have,

$$f(x) = \log\{ax^3 + (a+b)x^2 + (b+c)x + c\}$$

$$\Rightarrow f(x) = \log\{(ax^2 + bx + c)(x+1)\}$$

$$\Rightarrow f(x) = \log\left\{a\left(x + \frac{b}{2a}\right)^2 (x+1)\right\}$$

$$\Rightarrow f(x) = \log a + \log\left(x + \frac{b}{2a}\right)^2 + \log(x+1)$$

Since $a > 0$, therefore $f(x)$ is defined for $x \ne -\frac{b}{2a}$ and $x + 1 > 0$

i.e., $x \in R - \left\{\left\{-\frac{b}{2a}\right\} \cap (-\infty, -1)\right\}$

278 **(a)**

$$\because \quad y = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$$

$$\Rightarrow \quad \frac{y+1}{y-1} = \frac{10^x}{-10^{-x}}$$

[using componendo and dividendo rule]

$$\Rightarrow \quad 10^{2x} = \frac{1+y}{1-y}$$

$$\Rightarrow \quad 2x \log_{10} 10 = \log_{10}\left(\frac{1+y}{1-y}\right)$$

$$\Rightarrow \quad x = \frac{1}{2}\log_{10}\left(\frac{1+y}{1-y}\right)$$

$$\therefore \quad f^{-1}(x) = \frac{1}{2}\log_{10}\left(\frac{1+x}{1-x}\right)$$

279 **(b)**

Given, $\quad f(x) = \begin{cases} -1, \text{when } x \text{ is rational} \\ 1, \text{when } x \text{ is irrational} \end{cases}$

Now, $\quad (fof)(1-\sqrt{3}) = f[f(1-\sqrt{3})] = f(1) = -1$

280 **(c)**

We have,

$f(x) = 6^x + 6^{|x|} > 0$ for all $x \in R$

$\therefore$ Range $(f) \ne$ (Co $-$ domain (f))

So, $f: R \to R$ is an into function

For any $x, y \in R$, we find that

$$x \ne y \Rightarrow 2^x \ne 2^y \Rightarrow 2^{x+|x|} \ne 2^{y+|y|} \Rightarrow f(x) \ne f(y)$$

So, f is one-one

Hence, f is a one-one into function

281 **(a)**

Here, $Y = \{7, 11, \dots, \infty\}$

Let $\quad y = 4x + 3 \Rightarrow \frac{y-3}{4}$

Inverse of $f(x)$ is

$$g(y) = \frac{y-3}{4}$$

282 **(b)**

We have,

$$f(x) = \sqrt{\cos(\sin x)} + \sqrt{\sin(\cos x)}$$

We observe that $f(x)$ is not defined in $(\pi/2, 3\pi/2)$ and it is aperiodic function with period 2π. So, let us consider the internal $[-\pi/2, \pi/2]$ as it domain. Further, since $f(x)$ is an even function. So, we will consider $f(x)$ defined on $[0, \pi/2]$ only.

Clearly, $\sqrt{\cos(\sin x)}$ and $\sqrt{\sin(\cos x)}$ are decreasing functions on $[0, \pi/2]$

Range $(f) = \left[f\left(\frac{\pi}{2}\right), f(0)\right] = [\sqrt{\cos 1}, 1 + \sqrt{\sin 1}]$

284 **(c)**

We have,

$\log x > 1$ for all $x \in (e, \infty)$

$\Rightarrow \log(\log x) > 0$ for all $x \in (e, \infty)$

$\Rightarrow f(x) - \log[\log(\log x)] \in (-\infty, \infty)$ for all $x \in (e, \infty)$

Also, f is one-one. Hence, f is both one-one and onto

285 **(a)**

Given, $f(x) = x^2 - 3$

Now, $\quad f(-1) = (-1)^2 - 3 = -2$

$\Rightarrow \quad fof(-1) = f(-2) = (-2)^2 - 3 = 1$

$\Rightarrow \quad fofof(-1) = f(1) = 1^2 - 3 = -2$

Now, $\quad f(0) = 0^2 - 3 = -3$

$\Rightarrow \quad fof\,(0) = f(-3) = (-3)^2 - 3 = 6$

$\Rightarrow \quad fof(0) = f(6) = 6^2 - 3 = 33$

Again, $\quad f(1) = 1^2 - 3 = -2$

$\Rightarrow \quad fof(1) = f(-2) = (-2)^2 - 3 = 1$

$\Rightarrow \quad fofof(-1) + fofof(0) + fofof(1)$
$= -2 + 33 - 2 = 29$

Now, $f(4\sqrt{2}) = (4\sqrt{2})^2 - 3 = 32 - 3 = 29$

286 **(b)**

For any $x, y \in R$, we observe that

$f(x) = f(y) \Rightarrow \frac{x-m}{x-n} = \frac{y-m}{y-n} \Rightarrow x = y$

So, f is one-one

Let $\alpha \in R$ such that $f(x) = \alpha$

$\Rightarrow \frac{x-m}{x-n} = \alpha \Rightarrow x = \frac{m-n\alpha}{1-\alpha}$

Clearly, $x \in R$ for $\alpha = 1$. So, f is not onto

Hence, f is one-one into. This fact can also be observed from the graph of the function

287 **(b)**

We have,

$D(f) = R$ and $D(g) = R - \{0\}$

$\therefore D(h) = R - \{0\}$

Hence, $h(x) = f(x)g(x) = x \times \frac{1}{x} = 1$ for all $x \in R - \{0\}$

288 **(b)**

Since $\cos\sqrt{x}$ is not a periodic function. Therefore, $f(x) = \cos\sqrt{x} + \cos^2 x$ is not a periodic function

289 **(b)**

We have, $f(x) = 2^x$

$\therefore \frac{f(n+1)}{f(n)} = \frac{2^{n+1}}{2^n} = 2$ for all $n \in N$

Hence, $f(0), f(1), f(2), \ldots$ are in G.P.

290 **(d)**

We have,

$f(\sin x) - f(-\sin x) = x^2 - 1$ for all $x \in R$ ...(i)

Replacing x by $-x$, we get

$f(-\sin x) - f(\sin x) = x^2 - 1$ ...(ii)

Adding (i) and (ii), we get

$2(x^2 - 1) = 0 \Rightarrow x = \pm 1$

$\therefore x^2 - 2 = 1 - 2 = -1$

292 **(d)**

For $f(x)$ to be defined

$-1 \le \log_2 x \le 1 \qquad [\because -1 \le \sin^{-1} x \le 1]$

$\Rightarrow \quad \frac{1}{2} \le x \le 2$

293 **(a)**

We have,

$f(x) = |x|$ and $g(x) = [x]$

$\therefore g(f(x)) \le f(g(x))$

$\Rightarrow g(|x|) \le f([x]) \Rightarrow [|x|] \le |[x]|$

Clearly, $[|x|] = |[x]|$ for all $x \in Z$

Let $x \in (-\infty, 0)$ such that $x \notin Z$. Then, there exists positive integer k such that

$-k - 1 < x < -k$

$\Rightarrow [x] = -k - 1$ and $k < |x| < k + 1$

$\Rightarrow |[x]| = k + 1$ and $[|x|] = k$

$\Rightarrow [|x|] < |[x]|$

Hence, $[|x|] \le \left||x|\right|$ for all $x \in Z \cup (-\infty, 0)$

i.e. $\{x \in R : g(f(x)) \le f(g(x))\} = Z \cup (-\infty, 0)$

294 **(d)**

$\therefore f\left(\frac{3x+x^3}{1+3x^2}\right) - f\left(\frac{2x}{1+x^2}\right)$

$= \log\left(\frac{1 + \left(\frac{3x+x^3}{1+3x^2}\right)}{1 - \left(\frac{3x+x^3}{1+3x^2}\right)}\right) - \log\left(\frac{1 + \frac{2x}{1+x^2}}{1 - \frac{2x}{1+x^2}}\right)$

$= \log\left(\frac{1+x}{1-x}\right)^3 - \log\left(\frac{1+x}{1-x}\right)^2$

$= \log\left(\frac{1+x}{1-x}\right) = f(x)$

295 **(d)**

Clearly, $f(x)$ is defined if

$= \underbrace{\log_{10}\log_{10}\ldots\log_{10}}_{\rightarrow(n-1)\text{ times}\leftarrow} x > 0$

$\Rightarrow \underbrace{\log_{10}\ \log_{10}\ldots\log_{10}}_{(n-2)\text{ times}}\ x > 1$

$\Rightarrow \underbrace{\log_{10}\ \log_{10}\ldots\log_{10}}_{(n-3)\text{ times}}\ x > 10$

$\Rightarrow x > 10^{10^{10^{\cdot^{\cdot(n-2)\text{ times}}}}}$

Thus, domain of $f = \left(10^{10^{10^{\cdot^{\cdot(n-2)\text{ times}}}}}, \infty\right)$

296 **(a)**

Let $y = \sin^{-1}\left[\log_3\left(\frac{x}{3}\right)\right]$

$\Rightarrow \quad -1 \le \log_3\left(\frac{x}{3}\right) \le 1$

$\Rightarrow \quad \frac{1}{3} \le \frac{x}{3} \le 3$

$\Rightarrow \quad 1 \le x \le 9$

297 **(d)**

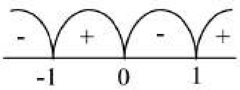
Since, $\quad f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$

For domain of $f(x)$,

$x^3 - 1 > 0, 4 - x^2 \neq 0)$

$\Rightarrow \quad x(x-1)(x+1) > 0$ and $x \neq \pm 2$

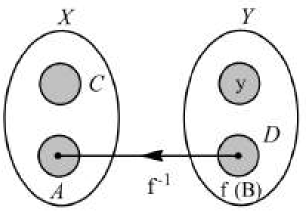
$\Rightarrow \quad x \in (-1, 0) \cup (1, \infty), \quad x \neq \pm 2$



$\Rightarrow \quad x \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

298 **(c)**

The given data is shown in the figure below



Since, $f^{-1}(D) = x$

$\Rightarrow \quad f(x) = D$

Now, if $B \subset X, f(B) \subset D$

$\Rightarrow \quad f^{-1}(f(B)) = B$

299 **(b)**

Clearly, $f(x)$ is an odd function

300 **(c)**

We have,

$$f(x) = \begin{cases} -1, -2 \le x \le 0 \\ x - 1, 0 \le x \le 2 \end{cases}$$

$\therefore f(|x|) = x \qquad [\because x \le 0]$

$\Rightarrow f(-x) = x$

$\Rightarrow -x - 1 = x \Rightarrow x = -\frac{1}{2}$

301 **(a)**

Given, $2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1 \qquad$ ...(i)

Replacing x by $\frac{1}{x}$, we get

$2f\left(\frac{1}{x^2}\right) + 3f(x^2) = \frac{1}{x^2} - 1 \qquad$ ...(ii)

On multiplying Eq. (i) by 2, Eq. (ii) by 3 and subtracting Eq. (i) from Eq. (ii), we get

$$5f(x^2) = \frac{3}{x^2} - 1 - 2x^2$$

$$\Rightarrow \quad f(x^2) = \frac{1}{5x^2}(3 - x^2 - 2x^4)$$

$$\Rightarrow \quad f(x^4) = \frac{1}{5x^4}(3 - x^4 - 2x^8)$$

[Replacing x by x^2]

$$= \frac{(1 - x^4)(2x^4 + 3)}{5x^4}$$

302 **(c)**

The function $f(x) = {}^{7-x}P_{x-3}$ is defined only if x is an integer satisfying the following inequalities:

(i)$7 - x \ge 0$ (ii)$x - 3 \ge 0$ (iii)$7 - x \ge x - 3$

Now,

$$\left.\begin{aligned} 7 - x \ge 0 &\Rightarrow x \le 7 \\ x - 3 \ge 0 &\Rightarrow x \ge 3 \\ 7 - x \ge x - 3 &\Rightarrow x \le 5 \end{aligned}\right\} \Rightarrow 3 \le x \le 5$$

Hence, the required domain is $\{3, 4, 5\}$

303 **(a)**

We have,

$f(x) = x, g(x) = |x|$ for all $x \in R$ and $\phi(x)$ satisfies the relation

$[\phi(x) - f(x)]^2 + [\phi(x) - g(x)]^2 = 0$

$\Rightarrow \phi(x) - f(x) = 0$ and $\phi(x) - g(x) = 0$

$\Rightarrow \phi(x) = f(x)$ and $\phi(x) = g(x)$

$\Rightarrow f(x) = g(x) = \phi(x)$

But, $f(x) = g(x) = x$, for all $x \ge 0$ $[\because |x| = x$ for all $x \ge 0]$

$\therefore \phi(x) = x$ for all $x \in [0, \infty)$

304 **(b)**

We observe that $f(x) = 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$ exists for

$$\frac{\pi^2}{16} - x^2 \ge 0 \Rightarrow -\frac{\pi}{4} \le x \le \frac{\pi}{4}$$

The least value of $\frac{\pi^2}{16} - x^2$ is 0 for $x = \pm\frac{\pi}{4}$ and the greatest value is $\frac{\pi^2}{16}$ for $x = 0$. Therefore, the greatest value of $f(x)$ occurs at $x = 0$ and the least value occurs at $x = \pm\pi/4$

Thus, greatest and least values of $f(x)$ are

$$f(0) = 3\sin\left(\sqrt{\frac{\pi^2}{16}}\right) = 3\sin\frac{\pi}{4} = \frac{3}{\sqrt{2}} \text{ and, } f\left(\frac{\pi}{4}\right)$$

$$= 3\sin 0 = 0$$

Hence, the value of $f(x)$ lie in the interval $[0, 3/\sqrt{2}]$

ALITER For $x \in [-\pi/4, \pi/4] = Dom\ (f)$, we find that $\sqrt{\frac{\pi^2}{16} - x^2} \in [0, \pi/4]$

Since $\sin x$ is an increasing function on $[0, \pi/4]$

$$\therefore \sin x \le \sin\sqrt{\frac{\pi^2}{16} - x^2} \le \sin\pi/4$$

$$\Rightarrow 0 \le 3\sin\sqrt{\frac{\pi^2}{16} - x^2} \le \frac{3}{\sqrt{2}} \Rightarrow 0 \le f(x) \le \frac{3}{\sqrt{2}}$$

305 **(b)**

$$f\left(\frac{\pi}{2} + x\right) = \left|\sin\left(\frac{\pi}{2} + x\right)\right| + \left|\cos\left(\frac{\pi}{2} + x\right)\right|$$

$$= |\cos x| + |\sin x| \text{ for all } x.$$

Hence, $f(x)$ is periodic with period $\frac{\pi}{2}$.

306 **(d)**

It can be easily checked that $g(x) = \left(\frac{x^{1/3} - b}{a}\right)^{1/2}$ satisfies the relation $fog\ (x) = gof\ (x)$

307 **(a)**

Since, $(1, 2) \in S$ but $(2, 1) \notin S$

$\therefore S$ is not symmetric.

Hence, S is not an equivalent relation.

Given, $T = \{(x, y): (x - y) \in I\}$

Now, $xTx \Rightarrow x - x = 0 \in I$, it is reflexive relation

Again, $xTy \Rightarrow (x - y) \in I$

$\Rightarrow y - x \in I \Rightarrow yTx$ it is symmetric relation.

Let xTy and yTz

$\therefore x - y = I_1$ and $y - z = I_2$

Now, $x - z = (x - y) + (y - z) = I_1 + I_2 \in I$

$\Rightarrow x - z \in I$

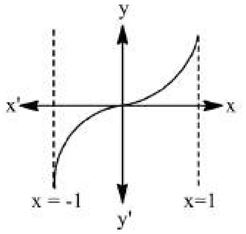
$\Rightarrow \ xTz$

$\therefore T$ is transitive.

Hence, T is an equivalent relation.

308 **(d)**

$f(x) = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$



Since, $-1 \leq x \leq 1$, therefore $-1 \leq f(x) \leq 1$

$\therefore$ Function is one-one onto.

309 **(c)**

We have,

$f(x) = \dfrac{1-x}{1+x}$

$\Rightarrow f(f(x)) = f\left(\dfrac{1-x}{1+x}\right) = \dfrac{1 - \frac{1-x}{1+x}}{1 + \frac{1-x}{1+x}} = x$

Again,

$f(x) = \dfrac{1-x}{1+x}$

$\Rightarrow f\left(\dfrac{1}{x}\right) = \dfrac{1 - \frac{1}{x}}{1 + \frac{1}{x}} = \dfrac{x-1}{x+1}$

$\therefore f\left(f\left(\dfrac{1}{x}\right)\right) = f\left(\dfrac{x-1}{x+1}\right) = \dfrac{1 - \frac{x-1}{x+1}}{1 + \frac{x-1}{x+1}} = \dfrac{1}{x}$

$\therefore \alpha = f(f(x)) + f\left(f\left(\dfrac{1}{x}\right)\right) = x + \dfrac{1}{x}$

$\Rightarrow |\alpha| = \left|x + \dfrac{1}{x}\right| \geq 2$

310 **(b)**

Let $A = \{1, 2, 3\}$

Let two transitive relations on the set A are

$R = \{(1,1), (1,2)\}$

And $\quad S = \{(2,2), (2,3)\}$

Now, $R \cup S = \{(1,1), (1,2), (2,2), (2,3)\}$

Here, $(1,2), (2,3) \in R \cup S \Rightarrow (1,3) \notin R \cup S$

$\therefore R \cup S$ is not transitive.

311 **(c)**

$f(1) = 3, f(2) = 4, f(3) = 5, f(4) = 6$

$\Rightarrow 1 \in B, 2 \in B$ do not have any pre-image in A

$\Rightarrow f$ is one-one and into

312 **(b)**

We observe that

$|f(x) + \phi(x)| = |f(x)| + |\phi(x)|$ is true, if

$f(x) \geq 0$ and $\phi(x) \geq 0$

OR

$f(x) < 0$ and $\phi(x) < 0$

$\Rightarrow (x > -1$ and $x > 2)$ or $(x < -1$ and $x < 2)$

$\Rightarrow x \in (2, \infty) \cup (-\infty, -1)$

313 **(b)**

We have, $f(x) = \dfrac{\sin^{-1}(3-x)}{\log_e(|x| - 2)}$

$\sin^{-1}(3-x)$ is defined for all x satisfying

$-1 \leq 3 - x \leq 1 \Rightarrow -4 \leq -x \leq -2 \Rightarrow x \in [2, 4]$

$\log_e(|x| - 2)$ is defined for all x satisfying

$|x| - 2 > 0 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$

Also, $\log_e(|x| - 2) = 0$ when $|x| - 2 = 1$ i.e., $x = \pm 3$

Hence, domain of $f = (2, 3) \cup (3, 4]$

314 **(a)**

$f(x)$ is defined

When $|x| > x$

$\Rightarrow \quad x < -x, x > x$

$\Rightarrow \quad 2x < 0, (x > x$ is not possible)

$\Rightarrow \quad x < 0$

Hence domain of $f(x)$ is $(-\infty, 0)$.

315 **(d)**

We have,

$f(x) = \log_{10}\{(\log_{10} x)^2 - 5(\log_{10} x) + 6\}$

Clearly, $f(x)$ assumes real values, if

$(\log_{10} x)^2 - 5\log_{10} x + 6 > 0$ and $x > 0$

$\Rightarrow (\log_{10} x - 2)(\log_{10} - 3) > 0$ and $x > 0$

$\Rightarrow (\log_{10} x < 2$ or $\log_{10} x > 3)$ and $x > 0$

$\Rightarrow (x < 10^2$ or, $x > 10^3)$ and $x > 0 \Rightarrow x \in (0, 10^2) \cup (10^3, \infty)$

316 **(b)**

We have,

$f\left(x + \dfrac{1}{x}\right) = x^2 + \dfrac{1}{x^2} = \left(x + \dfrac{1}{x}\right)^2 - 2$

$\Rightarrow f(y) = y^2 - 2$, where $y = x + \frac{1}{x}$

Now,

$x > 0 \Rightarrow y = x + \frac{1}{x} \geq 2$ and, $x < 0 \Rightarrow y = x + \frac{1}{x} \leq -2$

Thus, $f(y) = y^2 - 2$ for all y satisfying $|y| \geq 2$

317 **(c)**

Since $\sin x$ is a periodic function with period 2π and

$f(x) = \sin\left(\dfrac{2x+3}{6\pi}\right) = \sin\left(\dfrac{x}{3\pi} + \dfrac{1}{2\pi}\right)$

$\therefore f(x)$ is periodic with period $= \frac{2\pi}{1/3\pi} = 6\pi^2$

318 **(c)**

Let $f(x)=y$. Then,

$10x-7=y \Rightarrow x=\frac{y+7}{10} \Rightarrow f^{-1}(y)=\frac{y+7}{10}$

Hence, $f^{-1}(x)=\frac{x+7}{10}$

319 **(b)**

$\therefore\ f(2.5)=[2.5-2]=[0.5]=0$

320 **(c)**

We have,

$f(x)$

$=\sqrt{\log_{10}(\log_{10}x)-\log_{10}(4-\log_{10}x)-\log_{10}3}$

Clearly, $f(x)$ assumes real values, if

$\log_{10}(\log_{10}x)-\log_{10}(4-\log_{10}x)-\log_{10}3\geq 0$

$\Rightarrow \log_{10}\left\{\frac{\log_{10}x}{3(4-\log_{10}x)}\right\}\geq 0$

$\Rightarrow \frac{\log_{10}x}{3(4-\log_{10}x)}\geq 1$

$\Rightarrow \frac{4\log_{10}x-12}{3(4-\log_{10}x)}\geq 0$

$\Rightarrow \frac{\log_{10}x-3}{\log_{10}x-4}\leq 0$

$\Rightarrow 3\leq \log_{10}x<4 \Rightarrow 10^3\leq x<10^4 \Rightarrow x \in [10^3,10^4)$

Hence, domain of $f=[10^3,10^4)$

321 **(a)**

We observe that the periods of $\sin x$ and $\sin\frac{x}{n}$ are $\frac{2\pi}{|n|}$ and $2|n|\pi$ respectively

Therefore, $f(x)$ is periodic with period $2|n|\pi$

But, $f(x)$ has period 4π

$\therefore 2|n|\pi=4\pi \Rightarrow |n|=2 \Rightarrow n=\pm 2$

322 **(b)**

It can be easily checked that $f: R\to R$ given by $f(x)=\log_a(x+\sqrt{x^2+1})$ is a bijection

Now, $f(f^{-1}(x))=x$

$\Rightarrow \log_a\left(f^{-1}(x)+\sqrt{\{f^{-1}(x)\}^2+1}\right)=x$

$\Rightarrow f^{-1}(x)+\sqrt{\{f^{-1}(x)\}^2+1}=a^x$ ...(i)

$\Rightarrow \frac{1}{f^{-1}(x)+\sqrt{\{f^{-1}(x)\}^2+1}}=a^{-x}$

$\Rightarrow -f^{-1}(x)+\sqrt{\{f^{-1}(x)\}^2+1}=a^{-x}$ ...(ii)

Subtracting (ii) from (i), we get

$2f^{-1}(x)=a^x-a^{-x}$

$\Rightarrow f^{-1}(x)=\frac{1}{2}(a^x-a^{-x})$

323 **(d)**

We have,

$f(x)=x\frac{1+\frac{2}{\sqrt{x+4}}}{2-\sqrt{x+4}}+\sqrt{x+4}+4\sqrt{x+4}$

Clearly, $f(x)$ is defined for $x+4>0$ and $x\neq 0$

So, Domain of $f(x)$ is $(-4,0)\cup(0,\infty)$

324 **(d)**

$\because\ f(f(x))=f\left(\frac{\alpha x}{x+1}\right)$

$=\frac{\alpha\left(\frac{\alpha x}{x+1}\right)}{\left(\frac{\alpha x}{x+1}\right)+1}=\frac{\alpha^2x}{\alpha x+x+1}$

$\Rightarrow \frac{\alpha^2 x}{\alpha x+x+1}=x$

[given]

$\Rightarrow \alpha^2=\alpha x+x+1$

$\Rightarrow \alpha^2-1=(\alpha+1)x$

$\Rightarrow (\alpha+1)(\alpha-1-x)=0$

$\Rightarrow \alpha+1=0 \Rightarrow \alpha=-1$ $[\because \alpha-1-x\neq 0]$

325 **(d)**

$f(x)=\text{cosec}^2 3x+\cot 4x$

Period of $\text{cosec}^2 3x$ is $\frac{\pi}{3}$ and $\cot 4x$ is $\frac{\pi}{4}$.

$\therefore$ Period of $f(x)=$ LCM of $\left\{\frac{\pi}{3} \text{ and } \frac{\pi}{4}\right\}$

$=\frac{\text{LCM of }(\pi,\pi)}{\text{HCF of }(3,4)}=\frac{\pi}{1}=\pi$

326 **(b)**

Given, $f(x)=\sqrt{1+\log_e(1-x)}$

For domain, $(1-x)>0$ and $\log_e(1-x)\geq -1$

$\Rightarrow x<1$ and $1-x\geq e^{-1}$

$\Rightarrow x<1$ and $x\leq 1-\frac{1}{e}$

$\Rightarrow -\infty<x\leq\frac{e-1}{e}$

327 **(d)**

$\sin(\sin^{-1}x+\cos^{-1}x)=\sin\left(\frac{\pi}{2}\right)=1$

$\therefore$ Range of $\sin(\sin^{-1}x+\cos^{-1}x)$ is 1.

328 **(d)**

Given, $f(x)=\cos x-\sin x$

$=\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x\right)$

$=\sqrt{2}\cos\left(\frac{\pi}{2}+x\right)$

Since, $-1\leq\cos x\leq 1 \Rightarrow -1\leq\cos\left(\frac{\pi}{4}+x\right)\leq 1$

$\Rightarrow -\sqrt{2}\leq\sqrt{2}\cos\left(\frac{\pi}{4}+x\right)\leq\sqrt{2}$

$\therefore$ Range is $[-\sqrt{2},\sqrt{2}]$

329 **(a)**

Given, $f(x)=x^2+\frac{1}{x^2+1}$

$=(x^2+1)-\left(\frac{x^2}{x^2+1}\right)$

$=1+x^2\left(1-\frac{1}{x^2+1}\right)\geq 1, \forall x\in R$

Hence, range of $f(x)$ is $[1,\infty)$.

330 **(b)**

Let $y=\sqrt{\sin 2x} \Rightarrow 0 \le \sin 2x \le 1$,

$\Rightarrow \quad 0 \le 2x \le \frac{\pi}{2}$

$\Rightarrow \quad 0 \le x \le \frac{\pi}{4}$

$\Rightarrow \quad x \in \left[n\pi, n\pi+\frac{\pi}{4}\right]$

331 **(c)**

We have, $f(x)=x-[x]-\frac{1}{2}$

$\therefore f(x)=\frac{1}{2} \Rightarrow x-[x]=1$

But, for any $x \in R, 0 \le x-[x]<1$

$\therefore x-[x] \ne 1$ for any $x \in R$

Hence, $\left\{x \in R : f(x)=\frac{1}{2}\right\}=\phi$

332 **(c)**

Since, $x \in [-2,2]$, $x \le 0$ and $f(|x|)=x$

For $\quad -2 \le x \le 0$

$f(-x)=x \quad \Rightarrow \quad \le(-x)-1=x \quad \Rightarrow \quad x=-\frac{1}{2}$

333 **(d)**

Given, $\quad f(x)=\sin x$

And $\quad g(x)=\sqrt{x^2-1}$

$\therefore$ Range of $f=[-1,1] \notin$ domain of $g=(1,\infty)$

$\therefore gof$ is not defined.

334 **(d)**

Given, $f: C \to R$ such that $f(z)=|z|$

We know modulus of z and $\bar{z}$ have same values, so $f(z)$ has many one.

Also, $|z|$ is always non-negative real numbers, so it is not onto function.

335 **(b)**

We have,

$f(x)=\frac{x-1}{x+1}$

$\Rightarrow \frac{f(x)+1}{f(x)-1}=\frac{2x}{-2}$ [Applying componendo-dividendo]

$\Rightarrow x=\frac{f(x)+1}{1-f(x)}$

$\therefore f(2x)=\frac{2x-1}{2x+1}=\frac{2\left\{\frac{f(x)+1}{1-f(x)}\right\}-1}{2\left\{\frac{f(x)+1}{1-f(x)}\right\}+1}=\frac{3f(x)+1}{f(x)+3}$

336 **(b)**

Given, $f(x)=\tan\sqrt{\frac{\pi}{9}-x^2}$

For $f(x)$ to be defined $\frac{\pi^2}{9}-x^2 \ge 0$

$\Rightarrow \quad x^2 \le \frac{\pi^2}{9} \Rightarrow -\frac{\pi}{3} \le 3 \le \frac{\pi}{3}$

$\therefore$ Domain of $f=\left[-\frac{\pi}{3},\frac{\pi}{3}\right]$

The greatest value of $f(x)=\tan\sqrt{\frac{\pi^2}{9}-0}$, when $x=0$

And the least value of $f(x)=\tan\sqrt{\frac{\pi^2}{9}-\frac{\pi^2}{9}}$, when $x=\frac{\pi}{3}$

$\therefore$ The greatest value of $f(x)=\sqrt{3}$ and the least value of $f(x)=0$

$\therefore$ Range of $f=[0,\sqrt{3}]$.

337 **(b)**

We have,

$$[\sin x]=\begin{cases} 0, 0 \le x < \pi/2 \\ 1, x=\pi/2 \\ 0, \pi/2 < x \le \pi \\ -1, \pi < x < 2\pi \\ 0, x=\pi, 2\pi \end{cases}$$

And, $\text{cosec}^{-1}x$ is defined for $x \in (-\infty,-] \cup [1,\infty)$

$\therefore f(x)=\text{cosec}^{-1}[\sin x]$ is defined for $x=\frac{\pi}{2}$ and $x \in (\pi, 2\pi)$

Hence, domain of $\text{cosec}^{-1}[\sin x]$ is $(\pi,2\pi) \cup \left\{\frac{\pi}{2}\right\}$

338 **(a)**

aRa if $|a-a|=0<1$, which is true.

$\therefore$ It is reflexive.

Now, aRb,

$|a-b| \le 1 \Rightarrow |b-a| \le 1$

$\Rightarrow \quad aRb \Rightarrow bRa$

$\therefore$ It is symmetric.

339 **(b)**

Given

$f(x)=\log_e(x-[x])=\log_e\{x\}$

When x is an integer, then the function is not defined.

$\therefore$ Domain of the function $R-Z$.

340 **(b)**

Here, $f:[0,\infty] \to [0,\infty)$ *ie*, domain is $[0,\infty)$ and codomain is $[0,\infty)$.

For one-one $f(x)=\frac{x}{1+x}$

$\Rightarrow \quad f'(x)=\frac{1}{(1+x)^2}>0, \forall x \in [0,\infty)$

$\therefore f(x)$ is increasing in its domain. Thus, $f(x)$ is one-one in its domain.

For onto (we find range)

$f(x)=\frac{x}{1+x}$ *ie*, $y=\frac{x}{1+x} \Rightarrow y+yx=x$

$\Rightarrow x=\frac{y}{1-y} \Rightarrow \frac{y}{1-y} \ge 0$ as $x \ge 0 \therefore 0 \le y \ne 1$

ie, Range $\ne$ Codomain

$\therefore f(x)$ is one-one but not onto.

341 **(c)**

Given, $f(x)=x^3-1$

Let $x_1, x_2 \in R$

Now, $f(x_1) = f(x_2)$

$\Rightarrow \ x_1^3 - 1 = x_2^3 - 1$

$\Rightarrow \ x_1^3 = x_2^3$

$\Rightarrow \ x_1 = x_2$

$\therefore f(x)$ is one-one. Also, it is onto as range of $f = R$

Hence, it is a bijection.

342 **(d)**

Given $f(x) = [x]$ and $g(x) = |x|$

Now, $f\left(g\left(\frac{8}{5}\right)\right) = f\left(\frac{8}{5}\right) = \left[\frac{8}{5}\right] = 1$

And $g\left(f\left(-\frac{8}{5}\right)\right) = g\left(\left[-\frac{8}{5}\right]\right) = g(-2) = 2$

$\therefore \ f\left(g\left(\frac{8}{5}\right)\right) - g\left(f\left(-\frac{8}{5}\right)\right) = 1 - 2 = -1$

343 **(a)**

$\because \ f(x) = \dfrac{\cos^{-1} x}{[x]}$

For $f(x)$ to be defined $-1 \le x \le 1$ and $[x] \ne 0 \Rightarrow x \notin [0,1)$

$\therefore$ Domain of $f(x)$ is $[-1, 0) \cup \{1\}$.

344 **(c)**

Let $f(x) = g(x) + h(x) + u(x)$, where $g(x) = \frac{1}{x}, h(x) = 2^{\sin^{-1} x}$ and $u(x) = \frac{1}{\sqrt{x-2}}$

The domain of $g(x)$ is the set of all real numbers other than zero i.e. $R - \{0\}$

The domain of $h(x)$ is the set $[-1, 1]$ and the domain of $u(x)$ is the set of all reals greater than 2, i.e., $(2, \infty)$

Therefore, domain of $f(x) = R - \{0\} \cap [-1, 1] \cap (2, \infty) = \phi$

345 **(b)**

Given, $2f(x) + f(1-x) = x^2$ ...(i)

Replacing x by $(1-x)$, we get

$2f(1-x) + f(x) = (1-x)^2$

$\Rightarrow \ 2f(1-x) + f(x) = 1 + x^2 - 2x$ ...(ii)

On multiplying Eq. (i) by 2 and subtracting from Eq. (ii), we get

$$3f(x) = x^2 + 2x - 1 \Rightarrow f(x) = \frac{x^2 - 2x - 1}{3}$$

346 **(d)**

$f(x) = a + bx$

$\therefore \ f\{f(x)\} = a + b(a + bx) = a(1+b)b^2 x$

$\Rightarrow \ f[f\{f(x)\}] = f\{a(1+b) + b^2 x\}$

$= a(1 + b + b^2) + b^3 x$

$\therefore \ f^r(x)$

$= a(1 + b + b^2 + \dots + b^{r-1}) + b^r x$

$= a\left(\dfrac{b^r - 1}{b - 1}\right) + b' x$

347 **(b)**

We have,

$f(x) = \dfrac{x-1}{x+1}$

$\Rightarrow \dfrac{f(x)+1}{f(x)-1} = \dfrac{2x}{-2}$

$\Rightarrow x = \dfrac{f(x)+1}{1 - f(x)}$

$$\therefore f(2x) = \frac{2x-1}{2x+1} = \frac{2\left\{\frac{f(x)+1}{1-f(x)}\right\} - 1}{2\left(\frac{f(x)+1}{1-f(x)}\right) + 1} = \frac{3f(x)+1}{f(x)+3}$$

348 **(a)**

Since, $f(-x) = -f(x)$ and $f(x+2) = f(x)$

$\therefore \ f(x) = f(0)$ and $f(-2) = f(-2+2) = f(0)$

Now, $f(0) = f(-2) = -f(2) = -f(0)$

$\Rightarrow 2f(0) = 0 \ \Rightarrow \ f(0) = 0$

$\therefore \ f(4) = f(2) = f(0) = 0$

349 **(c)**

We observe that $\frac{1}{x^2-36}$ is not defined for $x = \pm 6$

Also, $\sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)}$ is a real number, if

$0 < \dfrac{x-1}{x++5} \le 1$

$\Rightarrow 0 < \dfrac{x-1}{x+5}$ and $\dfrac{x-1}{x+5} \le 1$

$\Rightarrow (x-1)(x+5) > 0$ and $1 - \dfrac{6}{x+5} \le 1$

$\Rightarrow (x < -5$ or $x > 1)$ and $-\dfrac{6}{x+5} \le 0$

$\Rightarrow (x < -5$ or $x > 1)$ and $x + 5 > 0$

$\Rightarrow (x < -5$ or $x > 1)$ and $x > -5$

Hence, domain of $f(x) = (1, \infty) - \{6\}$

350 **(b)**

Given, $f(x) = \log_2(\log_3(\log_4 x))$

We know, $\log_a x$ is defined, if $x > 0$

For $f(x)$ to be defined.

$\log_3 \log_4 x > 0, \log_4 x > 0$ and $x > 0$

$\Rightarrow \ \log_4 x > 3^0 = 1, x > 4^0 = 1$and $x > 0$

$\Rightarrow \ x > 4, x > 1$ and $x > 0$

$\Rightarrow \ \ x > 4$

351 **(c)**

We have,

$$f(x) = \begin{cases} -3x + 9, & \text{if } x < 2 \\ x - 3, & \text{if } 2 \le x < 3 \\ x - 1, & \text{if } 3 \le x < 4 \\ 3x - 9, & \text{if } x \ge 4 \end{cases}$$

$$\therefore g(x) = f(x+1) = \begin{cases} -3x + 6, & \text{if } x < 1 \\ x - 2, & \text{if } 1 \le x < 2 \\ x, & \text{if } 2 \le x < 3 \\ 3x - 6, & \text{if } x \ge 3 \end{cases}$$

Clearly, $g(x)$ is neither even nor odd. Also, $g(x)$ is not a periodic function

352 **(b)**

We have,

$f:[2,\infty)\to B$ such that $f(x)=x^2-4x+5$

Since f is a bijection. Therefore, $B=$ Range of f

Now,

$f(x)=x^2-4x+5=5=(x-2)^2+1$ for all $x\in[2,\infty)$

$\Rightarrow f(x)\geq 1$ for all $x\in[2,\infty)\Rightarrow$ Range of $f=[1,\infty)$

Hence, $B=[1,\infty)$

353 **(d)**

Given, $R=\{(x,y):4x+3y=20\}$.

Since, R is a relation on N, therefore x,y are the elements of N. But in options (a) and (b) elements are not natural numbers and option (c) does not satisfy the given relation $4x+3y=20$.

354 **(b)**

Since the function $f:R\to R$ given by $f(x)=x^3+5$ is a bijection. Therefore, f^{-1} exists

Let $f(x)=y$. Then,

$x^3+5=y$

$\Rightarrow x=(y-5)^{1/3}$ $\quad[\because f(x)=y\Leftrightarrow x=f^{-1}(y)]$

Hence, $f^{-1}(x)=(x-5)^{1/3}$

355 **(a)**

We have,

$f(x)=x, g(x)=|x|$ for all $x\in R$

Now,

$[\phi(x)-f(x)]^2+[\phi(x)-g(x)]^2=0$

$\Rightarrow \phi(x)-f(x)=0$ and $\phi(x)-g(x)=0$

$\Rightarrow \phi(x)=f(x)$ and $\phi(x)=g(x)$

$\Rightarrow f(x)=g(x)=\phi(x)$

But, $f(x)=g(x)=x$, for all $x\geq 0$ $[\because |x|=x$ for all $x\geq 0]$

$\therefore \phi(x)=x$ for all $x\in[0,\infty)$

356 **(b)**

Since $f(x)$ is defined for $x\in[0,1]$. Therefore, $f(2x+3)$ exists if

$0\leq 2x+3\leq 1\Rightarrow -\frac{3}{2}\leq x\leq -1\Rightarrow x\in[-3/2,-1]$

358 **(a)**

$fog(-1)=f\{g(-1)\}$

$=f(-7)=5-49=-44$

359 **(a)**

We have,

$f(x)=\frac{e^{x^2}-e^{-x^2}}{e^{x^2}+e^{-x^2}}$ for all $x\in R$

Clearly, $f(-x)=f(x)$ for all $x\in R$

So, f is a many-one function

Also, $e^{x^2}>e^{-x2}>0$

So, $f(x)$ attains only positive values

Consequently, range of $\neq R$

Hence, f is many-one into function

360 **(c)**

Let $x,y\in N$ such that $f(x)=f(y)$

$\Rightarrow\ x^2+x+1=y^2+y+1$

$\Rightarrow\ (x-y)(x+y+1)=0$

$\Rightarrow\ x=y$ or $x=(-y-1)\notin N$

$\therefore f$ one-one.

Also, f is not onto.

361 **(c)**

The period of the function in option (a) is 2. The period of the function in option (b) is 24.

The period of the function in option (c) is 2π.

362 **(a)**

We have,

$f(x)=\sqrt{3}\sin x+\cos x+4$

$\Rightarrow f(x)=2(\sin x\cos\pi/6+\cos x\sin\pi/6)+4$

$\Rightarrow f(x)=2\sin(x+\pi/6)+4$

Clearly, $f(x)$ will be a bijection, if $\sin(x+\pi/6)$ is a bijection

Now,

$\sin(x+\pi/6)$ is a bijection

$\Rightarrow -\pi/2\leq x+\pi/6\leq\pi/2$

$\Rightarrow -2\pi/3\leq x\leq\pi/3$

$\Rightarrow x\in[-2\pi/3,\pi/3]$

For $x\in[-2/3\pi,\pi/3]$, we have

$-1\leq\sin(x+\pi/6)\leq 1$

$\Rightarrow -2\leq 2\sin(x+\pi/6)\leq 2$

$\Rightarrow -2+4\leq 2\sin(x+\pi/6)+4\leq 2+4$

$\Rightarrow 2\leq f(x)\leq 6$

$\Rightarrow$ Range of $f(x)=[2,6]$

Hence, $A=[-2\pi/3,\pi/3]$ and $B=[2,6]$

363 **(c)**

We have,

$f(x)=2x+3$ and $g(x)=x^2+7$

$\therefore g(f(x))=g(2x+3)=(2x+3)^2+7$

Now,

$g(f(x))=8$

$\Rightarrow (2x+3)^2+7=8$

$\Rightarrow (2x+3)^2=1$

$\Rightarrow 2x+3=\pm1\Rightarrow 2x=-4,-2\Rightarrow x=-1,-2$

364 **(c)**

We have,

$f(x)=\sin^{-1}\left(\frac{x-3}{2}\right)-\log(4-x)=g(x)+h(x)$

where $g(x) = \sin^{-1}\left(\frac{x-3}{2}\right)$ and $h(x)$

$= -\log(4-x)$

now, $g(x)$ is defined for

$-1 \le \frac{x-3}{2} \le 1 \Rightarrow -2 \le x-3 \le 2 \Rightarrow 1 \le x \le 5$

and, $h(x)$ is defined for $4 - x > 0 \Rightarrow x < 4$

So, domain of $f(x) = [1,5] \cap [-\infty, 4) = [1,4)$

365 **(a)**

Let $y = f(x) = \frac{1-x}{1+x}$ $[\because x \ne -1]$

$\Rightarrow \quad x = \frac{1-y}{1+y}$

$\therefore \quad f^{-1}(x) = \frac{1-x}{1+x} = f(x)$

366 **(b)**

Since, $3f(x) + 2f\left(\frac{x+59}{x-1}\right) = 10x + 30$ ... (i)

Replacing x by $\frac{x+59}{x-1}$ in Eq. (i), we get

$\therefore \quad 3\left(\frac{x+59}{x-1}\right) + 2f(x) = \frac{40x+560}{x-1}$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$f(x) = \frac{6x^2 - 4x - 242}{x-1}$$

$$\therefore \quad f(7) = \frac{6 \times 49 - 28 - 242}{6} = 4$$

367 **(c)**

$$\left[\frac{2}{3} + \frac{r}{99}\right] = \begin{cases} 0, & r < 33 \\ 1, & r \ge 33 \end{cases}$$

$$\therefore \quad \sum_{r=0}^{98}\left[\frac{2}{3} + \frac{r}{99}\right] = \sum_{r=0}^{32}\left[\frac{2}{3} + \frac{r}{99}\right] + \sum_{r=33}^{98}\left[\frac{2}{3} + \frac{r}{99}\right]$$

$$= 0 + 66 = 66$$

368 **(b)**

We have, Domain $(f) = [0,1]$

$\therefore f(3x^2)$ is defined, if

$0 \le 3x^2 \le 1$

$\Rightarrow 0 \le x^2 \le \frac{1}{3} \Rightarrow |x| \le \frac{1}{\sqrt{3}} \Rightarrow x \in [-1/\sqrt{3}, 1/\sqrt{3}]$

369 **(d)**

$\sin x - \sqrt{3}\cos x = 2\sin\left(x - \frac{\pi}{3}\right)$

Since, $\quad -2 \le 2\sin\left(x - \frac{\pi}{3}\right) \le 2$

$\Rightarrow \quad -1 \le 1 + 2\sin\left(x - \frac{\pi}{3}\right) \le 3$

$\therefore$ Range of $S = [-1, 3]$

370 **(b)**

Given,

$$f(x) = e^x \quad \text{and} \quad g(x) = \log_e x$$

Now, $\quad f\{g(x)\} = e^{\log_e x} = x$

And $\quad g\{f(x)\} = \log_e e^x = x$

$\therefore \quad f\{g(x)\} = g\{f(x)\}$

371 **(a)**

The function $f(x) = {}^{7-x}P_{x-3}$ is defined only if x is an integer satisfying the following inequalities:

(i)$7 - x \ge 0$ (ii)$x - 3 \ge 0$ (iii)$7 - x \ge x - 3$

Now,

$$\left.\begin{array}{r} 7 - x \ge 0 \Rightarrow x \le 7 \\ x - 3 \ge 0 \Rightarrow x \ge 3 \\ 7 - x \ge x - 3 \Rightarrow x \le 5 \end{array}\right\} \Rightarrow 3 \le x \le 5$$

Hence, the required domain is $\{3, 4, 5\}$

Now,

$f(3) = {}^{7-3}P_0, f(4) = {}^{3}P_1 = 3$ and $f(5) = {}^{2}P_2 = 2$

Hence, range of $f = \{1, 2, 3\}$

372 **(c)**

We have,

$f(x) = \log_{1.7}\left\{\frac{2 - \varphi'(x)}{x+1}\right\}$, where $\varphi(x)$

$$= \frac{x^3}{3} - \frac{3}{2}x^2 - 2x + \frac{3}{2}$$

For $f(x)$ to be defined, we must have

$\frac{2 - \varphi'(x)}{x+1} > 0, x \ne -1$

$\Rightarrow \frac{2 - (x^2 - 3x - 2)}{3x + 1} > 0, x \ne -1$

$\Rightarrow \frac{x^2 - 3x - 4}{x+1} < 0, x \ne -1$

$\Rightarrow \frac{(x-4)(x+1)}{x+1} < 0, x \ne -1$

$\Rightarrow x - 4 < 0, x \ne -1$

$\Rightarrow x < 4, x \ne -1$

$\Rightarrow x \in (-\infty, 4), x \ne -1 \Rightarrow x \in (-\infty, -1) \cup (-1, 4)$

373 **(a)**

$f(x)$ is defined, if

$-1 \le \frac{4}{3 + 2\cos x} \le 1$

$\Rightarrow \frac{4}{3 + 2\cos x} \le 1$ $[\because 3 + 2\cos x > 0]$

$\Rightarrow 4 \le 3 + 2\cos x$

$\Rightarrow \cos x \ge \frac{1}{2} \Rightarrow 2n\pi - \frac{\pi}{6} \le x \le \frac{\pi}{6}, n \in Z$

374 **(c)**

The period of the function in (a) is 2. The period of the function in (b) is 24. The period of the function in (c) is 2π

375 **(a)**

$R = \{(a, b): 1 + ab > 0\}$

It is clear that the given relation on S is reflexive, symmetric but not transitive.

377 **(a)**

We have,

$f(x) = \max\{(1 - x), 2, (1 + x)\}$

For $x \le -1$, we find that

$1 - x \geq 2$, and $1 - x \geq 1 + x$

$\therefore$ Max$\{(1-x), 2, (1+x)\} = 1 - x$

For $-1 < x < 1$, we find that

$0 < 1 - x < 2$, and $0 < 1 + x < 2$

$\therefore$ Max$\{(1-x), (1+x)\} = 2$

For $x \geq 1$, we observe that

$1 + x \geq 2, 1 + x > 1 - x$

$\therefore$ Max$\{(1-x), 2, (1+x)\} = 1 + x$

Hence, $f(x) = \begin{cases} 1-x, & x \leq -1 \\ 2, & -1 < x < 1 \\ 1+x, & x \geq 1 \end{cases}$

NOTE

Students are advised to solve this problem by d

$y = 1 - x, y = 2$ and $y = 1 + x$

378 **(d)**

Period of $\sin\frac{\theta}{3} = 6\pi$

And period of $\cos\frac{\theta}{2} = 4\pi$

$\therefore$ Period of $f(x) = \text{LCM}(6\pi, 4\pi) = 12\pi$

379 **(b)**

To make $f(x)$ an odd function in the interval $[-1,1]$, we re-define $f(x)$ as follows:

$f(x) = \begin{cases} f(x), & 0 \leq x \leq 1 \\ -f(-x), & -1 \leq x < 0 \end{cases}$

$\Rightarrow f(x)$

$= \begin{cases} x^2 + x + \sin x - \cos x + \log(1+|x|), & 0 \leq \\ -(x^2 - x - \sin x - \cos x + \log(1+|x|), & -1 \end{cases}$

$\Rightarrow f(x)$

$= \begin{cases} x^2 + x + \sin x - \cos x + \log(1+|x|), 0 \leq x \leq \\ -x^2 + x + \sin x + \cos x - \log(1+|x|), -1 \leq x \end{cases}$

Thus, the odd extension of $f(x)$ to the interval $[-1, 1]$ is

$-x^2 + x + \sin x + \cos x - \log(1+|x|)$

380 **(b)**

We have,

$g(x) = 1 + \sqrt{x}$ and $f(g(x)) = 3 + 2\sqrt{x} + x$

Now,

$f(g(x)) = 3 + 2\sqrt{x} + x$

$\Rightarrow f(g(x)) = 2 + (1 + \sqrt{x})^2$

$\Rightarrow f(g(x)) = 2 + \{g(x)\}^2$

$\Rightarrow f(x) = 2 + x^2$

381 **(a)**

Given, $f(x) = \tan^{-1}\frac{2x}{1-x^2} = 2\tan^{-1}x (x^2 < 1)$

Since, $x \in (-1, 1)$.

$\Rightarrow \tan^{-1}x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

$\Rightarrow 2\tan^{-1}x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

So, $f(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

382 **(a)**

Let $y = f(x) = x^3$

$\therefore \quad x = y^{1/3}$

$\Rightarrow \quad f^{-1}(x) = x^{1/3}$

$\therefore \quad f^{-1}(8) = (8)^{1/3} = 2$

383 **(d)**

For $f(x) = \log_{\left(\frac{x-2}{x+3}\right)} 2$ to exist, we must have

$\frac{x-2}{x+3} > 0$ and $\frac{x-2}{x+3} \neq 1 \Rightarrow x < -3$ or $x > 2, x \neq -3, x \neq 2$

For $g(x) = \frac{1}{\sqrt{x^2-9}}$ to exist, we must have

$x^2 - 9 > 0 \Rightarrow x < -3$ or $x > 0$

Thus, $f(x)$ and $g(x)$ both do not exist for $-3 < x < 2$, i.e., for $x \in (-3, 2)$

384 **(b)**

For choice (a), we have

$f(x) = f(y), x, y \in [-1, \infty)$

$\Rightarrow |x+1| = |y+1| \Rightarrow x+1 = y+1 \Rightarrow x = y$

So, f is an injection

For choice (b), we have

$g(2) = \frac{5}{2}$ and $g(1/2) = \frac{5}{2}$

$\therefore 2 \neq \frac{1}{2}$ but $g(2) = g(1/2)$

Thus, $g(x)$ is not injective

It can be easily seen that choices $h(x)$ and $k(x)$ are injections

385 **(b)**

We have

$f(n) = \begin{cases} 2 & \text{if } n = 3k, & k \in Z \\ 10 & \text{if } n = 3k+1, & k \in Z \\ 0 & \text{if } n = 3k+2, & k \in Z \end{cases}$

For $f(n) > 2$, we take $n = 3k + 1, k \in Z$

$\Rightarrow \quad n = 1, 4, 7$

$\therefore$ Required set $\{n \in Z; f(n) > 2\} = \{1, 4, 7\}$

386 **(b)**

Let $\quad y = \frac{2x-1}{x+5}$

$\Rightarrow \quad x = \frac{5y+1}{2-y}$

$\therefore f^{-1}(x) = \frac{5x+1}{2-x}, x \neq 2$

387 **(b)**

We have,

$f(a+x) = b + [b^3 + 1 - 3b^2 f(x) + 3b\{f(x)\}^2 - \{f(x)\}^3]^{1/3}$ for all $x \in R$

$\Rightarrow f(a+x) = b + [1 + \{b - f(x)\}^3]^{1/3}$ for all $x \in R$

$\Rightarrow f(a+x) - b = [1 - \{f(x) - b\}^3]^{1/3}$ for all $x \in R$

$\Rightarrow g(a+x) = [1 - \{g(x)\}^3]^{1/3}$ for all $x \in R$, Where $g(x) = f(x) - 1$

$\Rightarrow g(2a + x) = [1 - \{g(a + x)\}^3]^{1/3}$ for all $x \in R$

$\Rightarrow g(2a + x) = \left[1 - \left\{1 - \left(g(x)\right)^3\right\}\right]^{1/3}$ for all $x \in R$

$\Rightarrow g(2a + x) = g(x)$ for all $x \in R$

$\Rightarrow f(2a + x) - 1 = f(x) - 1$ for all $x \in R$

$\Rightarrow f(2a + x) = f(x)$ for all $x \in R$

$\Rightarrow f(x)$ is periodic with period $2a$

388 **(a)**

Given a set containing 10 distinct elements and $f: A \to A$ Now, every element of a set A can make image in 10 ways.

$\therefore$ Total number of ways in which each element make images $= 10^{10}$.

389 **(c)**

Given, $f\left(\frac{p}{q}\right) = \sqrt{p^2 - q^2}$, for $\frac{p}{q} = Q$

If $p < q$, then $f\left(\frac{p}{q}\right)$ is not real.

Hence, statement I is false while statement II is true.

390 **(c)**

The given function is defined when $x^2 - 1; 3 + x > 0$ and $3 + x \neq 1$

$\Rightarrow \quad x^2 > 1; \quad 3 + x > 0 \quad$ and $x \neq -2$

$\Rightarrow \quad -1 > x > 1; \quad x > -3, \quad x \neq -2$

$\therefore$ Domain of the function is

$D_f = (-3, -2) \cup (-2, -1) \cup (1, \infty)$

391 **(a)**

Let x and y be two arbitary elements in A.

Then, $f(x) = f(y)$

$$\Rightarrow \quad \frac{x - 2}{x - 3} = \frac{y - 2}{y - 3}$$

$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$

$\Rightarrow x = y, \forall\, x, y \in A$

So, f is an injective mapping.

Again, let y be an orbitary element in B, then

$$f(x) = y$$

$$\Rightarrow \quad \frac{x - 2}{x - 3} = y$$

$$\Rightarrow \quad x = \frac{3y - 2}{y - 1}$$

Clearly, $\forall\, y \in B, x = \frac{3y-2}{y-1} \in A$, thus for all $y \in B$ there exists $x \in A$ such that

$$f(x) = f\left(\frac{3y - 1}{y - 1}\right) = \frac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3} = y$$

Thus, every element in the codomain B has its preimage in A, so f is a surjection. Hence, $f: A \to B$ is bijective.

392 **(a)**

$f(x)$ is defined for

$\sin x \geq 0$ and $1 + \sqrt[3]{\sin x} \neq 0$

$\Rightarrow \sin x \geq 0$ and $\sin x \neq -1$

$\Rightarrow \sin x \geq 0$

$\Rightarrow x \in [2n\,\pi, (2n + 1)\pi], n \in Z$

$\Rightarrow D = \bigcup_{n \in Z} [2n\,\pi, (2n + 1)\pi]$

Clearly, it contains the interval $(0, \pi)$

393 **(a)**

$$fog\,(x) = f\big(g(x)\big) = f(3x - 1) = 3(3x - 1)^2 + 2$$
$$= 27x^2 - 18x + 5$$

394 **(c)**

We have,

$$|x| = \begin{cases} x, & x \geq 0 \\ x, & x < 0 \end{cases} \Rightarrow |x| - x = \begin{cases} 0, & x \geq 0 \\ -2\,x, & x < 0 \end{cases}$$

Hence, domain of $f(x) \frac{1}{\sqrt{|x| - x}}$ is the set of all negative real numbers, i.e., $(-\infty, 0)$

396 **(c)**

$$gof(x) = g\{f(x)\}$$
$$= g(x^2 - 1) = (x^2 - 1 + 1)^2$$
$$= x^4$$

397 **(d)**

$$\sum_{r=1}^{n} f(r) = f(1) + f(2) + f(3) + \dots + f(n)$$
$$= f(1) + 2f(1) + 3f(1) + \dots nf(n)$$
$$[\text{since, } f(x + y) = f(x) + f(y)]$$
$$= (1 + 2 + 3 + \dots + n)f(1) = f(1) \sum n$$
$$= \frac{7n(n + 1)}{2} \qquad [\because f(1) = 7 \text{ (given)}]$$

398 **(c)**

Given, $f(x) = 2x^4 - 13x^2 + ax + b$ is divisible by $(x - 2)(x - 1)$

$\therefore \quad f(2) = 2(2)^4 - 13(2)^2 + a(2) + b = 0$

$\Rightarrow \quad 2a + b = 20 \qquad$ ... (i)

And $\quad f(1) = 2(1)^4 - 13(1)^2 + a + b = 0$

$\Rightarrow \quad a + b = 11 \qquad$ ... (ii)

On solving Eqs. (i) and (ii), we get

$a = 9, \quad b = 2$

399 **(d)**

We have, $f(x) = \frac{x^2-8}{x^2+2}$

Clearly, $f(-x) = f(x)$. Therefore, f is not one-one

Again,

$$f(x) = \frac{x^2 - 8}{x^2 + 2} = 1 - \frac{10}{x^2 + 2}$$

$\Rightarrow f(x) < 1 \quad$ for all $x \in R$

$\Rightarrow$ Range $f \neq$ Co-domain of f $i.e.\ R$.

So, f is not onto. Hence, f is neither one-one nor onto

400 **(b)**

$\sin^{-1}(x-3)$ is defined for the values of x satisfying

$-1 \leq x-3 \leq 1 \Rightarrow 2 \leq x \leq 4 \Rightarrow x \in [2,4]$

$\sqrt{9-x^2}$ is defined for the values of x satisfying

$9-x^2 \geq 0 \Rightarrow x^2-9 \leq 0 \Rightarrow x \in [-3,3]$

Also, $\sqrt{9-x^2}=0 \Rightarrow x=\pm 3$

Hence, the domain of $f(x)$ is $[2,4] \cap [-3,3] - \{-3,3\} = [2,3)$